Reverse Bonus Certificates

In markets with a negative long-term trend (bear markets) investments that "go with the market" (such as shares, bonus certificates or discount certificates) are not beneficial. Investments that behave inversely to the market, such as reverse bonus certificates, are quite different: you gain with falling or constant prices of the underlying asset and you lose with strong price increases. As in the case of bonus certificates there are two free parameters to choose, the absorbing barrier and the bonus level. In addition, there is a further parameter, called the reflection point, which different issuers call by different names, adding to the confusion. It can be shown that reverse bonus certificates have the same spectrum of return distribution functions as bonus certificates. Overall however, choosing the right reverse bonus certificate is more difficult than choosing a bonus certificate. Therefore, calculating the gain and loss probabilities at expiration date and calculating the fair price at purchase time become even more important in making a purchase decision.

Strengths of reverse bonus certificates:

- If the absorbing barrier and the bonus level are chosen appropriately, it is possible to have positive returns for a wide range of underlying asset prices, just as with bonus certificates.
- If the underlying asset price falls below the bonus level, the profit increases linearly with decreasing underlying asset price.
- With the right combination of absorbing barrier A and bonus levels B you can achieve the same range of investment opportunities as with a bonus certificate. Each parameter pair (A, B) leads to a return distribution function that is similar or equal to the distribution function of a fixed term deposit, or a bet with varying odds, or the underlying asset itself.

Weaknesses of reverse bonus certificates:

- If the underlying asset touches or breaks through the barrier during the lifetime of the certificate, the loss is usually very high.
- Due to the complicated structure of reverse bonus certificates, the right choice of parameters (barrier, bonus level and reflection point) is difficult.
- The number and variety of reverse bonus certificates on the market is smaller than that of discount and bonus certificates. In addition, they are less transparent due to their more complicated design. In practice this means that often reverse bonus certificates are more expensive than their fair price.

A **theoretical description of reverse bonus certificates** can be found in:

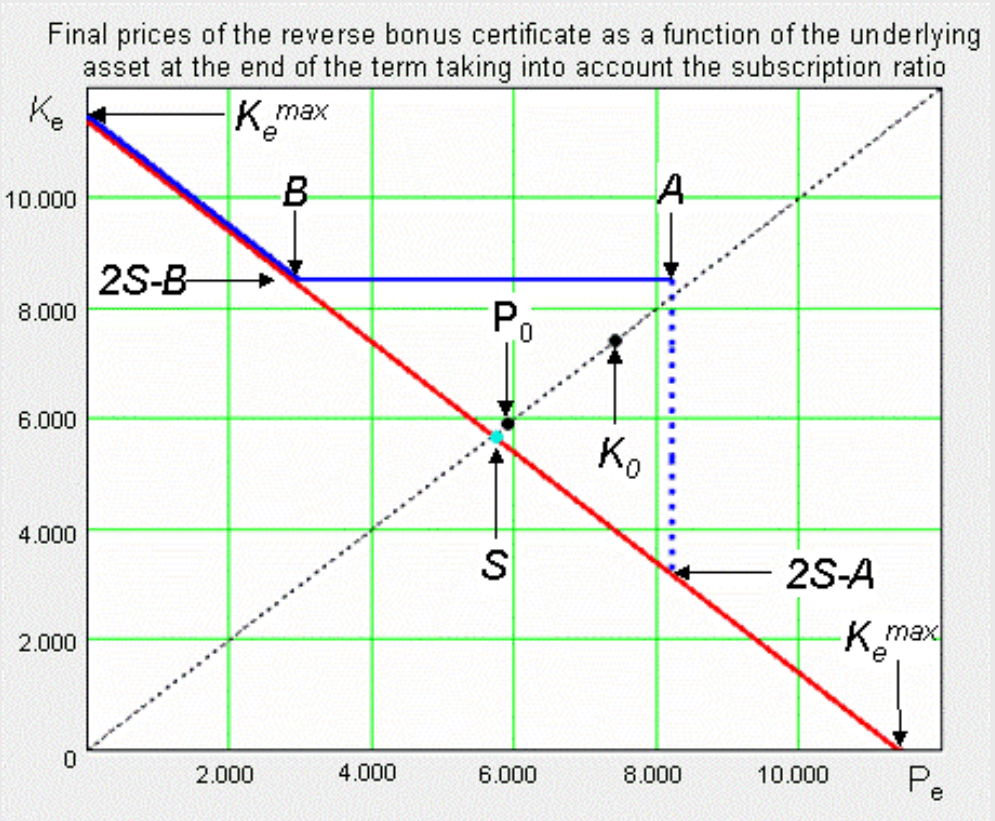
http://www.sigmadewe.com/fileadmin/user_upload/pdf-Dateien/Theorie_ReverseBonuszertifikate.pdf (in German)

Reverse Bonus Certificates

1. Design of reverse bonus certificates

Like the bonus certificate the reverse bonus certificate is determined by two parameters, the absorbing barrier *A* and the bonus level *B*. While the price of a bonus certificate at expiration theoretically has no upper limit (because the underlying asset has no upper limit), a reverse bonus certificate can only assume a maximum value K_e^{max}. This value occurs when the price of the underlying asset has fallen to zero. It is linked to the reflection point *S* by the relation $K_e^{max} = 2 \cdot S$.

The possible prices, K_e, of the reverse bonus certificate at expiration can be read off from the graph below. In this real example, the underlying asset is the German DAX index with a price P_0=5918 at purchase time. The parameters chosen for the absorbing barrier and the bonus level are: A = 8200, B = 3000. The reflection point is at S = 5750. Taking into account the subscription ratio of 0.01, the purchase price of the reverse bonus certificate is K_0 = 7418. The certificate's term is 422 days.



Reverse Bonus Certificates

1. Design of reverse bonus certificates

Blue curve: final prices K_e of the reverse bonus certificate if the underlying asset has always remained below A during the certificate's entire lifetime. The maximum possible final price K_e^{max} = 11500 is obtained when the price of the underlying asset is zero.

Red curve: final prices K_e of the reverse bonus certificate if the price of the underlying asset has touched or crossed the barrier A at least once during the certificate's term. In this case, all values of K_e lie exclusively on the red curve. Total loss occurs when the price of the underlying asset lies above K_e^{max} = 11500 on expiry of the certificate.

Black curve: prices of the underlying asset.

Results:

As in the case of bonus certificates, the return, $R = K_e/K_0 - 1$, of the reverse bonus certificate at expiration, not only depends on the final price, P_e, of the underlying asset, but on the entire price history between issue and expiration time. The investor gets the bonus return, $R = (2 \cdot S - B)/K_0 - 1$, when K_e is between B and A and the underlying asset has never touched or crossed the barrier A during the certificate's term. In this example R amounts to 14.6%.

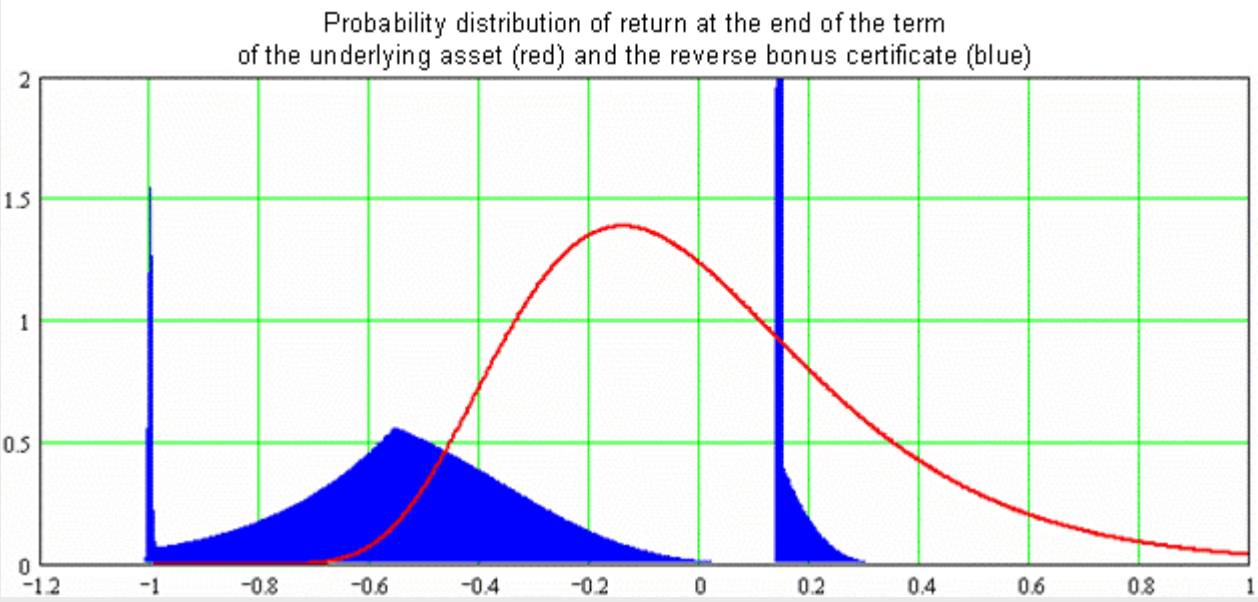
Reverse Bonus Certificates

2. Gain and loss probabilities, fair price, meaning of S

In the article **Theory of Reverse Bonus Certificates** (in German) it is shown that the statistical properties of a reverse bonus certificate can be expressed by those of a corresponding bonus certificate. With the help of the probability distribution of return, i.e. the probability of getting a certain return at the time of the certificate's expiration, one can calculate all relevant variables, such as the profit and loss probabilities, the average return, the volatility and the fair price. By mapping the reverse bonus certificate to a corresponding bonus certificate one gets the same return distribution patterns as in the **parameter matrix** for bonus certificates.

Blue curve: probability distribution of return of the reverse bonus certificate at its expiration. With the same values for the bonus level and the absorbing barrier as the above example, one gets a split of the distribution function into two distinct peaks whose maxima are determined by the relative positions of P_0 , K_0 , S, B, and A. The non-negligible additional peak at R = -1 is caused by all the cases of total loss, when the price of the underlying asset is larger than or equal K_e^{max} at the certificate's expiration.

Red curve: probability distribution of return of the underlying asset at the time of the reverse bonus certificate's expiration. This curve is generated by applying the Black Scholes model to the DAX index using adequate parameters for the average annual return and the annual volatility. Realistic values for a virtual purchase time of mid-October 2011 are 0% for the average annual return and 30% for its volatility.



Reverse Bonus Certificates

2. Gain and loss probabilities, fair price, meaning of S

Results:

- The probability distribution of return for the chosen reverse bonus certificate corresponds to that of a bet. Either you gain at least the reverse bonus return of 14.6% with probability p_g = 74% (the bet is won), or, with probability p_l = 25.1%, you suffer a heavy loss with the loss peak at $R = (2 \cdot S - A)/K_0 - 1$ = -55.5% (the case $R = -1$ is ignored). Losses equal and higher than -55.5% occur with a probability of 11.3%. If you had invested directly in the underlying asset, the probability of such losses would be only 0.8%. However, the probability of total loss would be more than twice as high: p_l = 56.3%.
- The fair price of this reverse bonus certificate is K_f = 7046, taking the subscription ratio into account and assuming a risk-free annual return of 2%. This makes the purchase price that is fixed by the issuer 5.3% higher than the fair price. This overpricing was typical for reverse bonus certificates that were offered in mid-October 2011. In contrast, the fair prices of bonus and discount certificates (offered at the same time) on the underlying DAX index were close to or even higher than market prices.
- The location of the reflection point *S* relative to the starting point P_0 of the underlying asset determines the location and the width of the profit and loss peaks (keeping *A* and *B* fixed): The higher the ratio S / P_0 is, the better the starting position for a successful bet is, since P_0 is further away from the absorbing barrier *A*. At the same time, a larger ratio of S / P_0 leads to a higher fair price with the result that when the certificate expires both the average return and the volatility of the reverse bonus certificate are smaller than for a small S / P_0 ratio (see **Theory of Reverse Bonus Certificates**).