Bonus Certificates

Suppose you are currently in the stock market because of positive market signals. You want to back up your portfolio against moderate market corrections by means of certificates. At the same time you want to participate in profits if the market rises strongly against your current expectations. Bonus certificates often are the more profitable alternative to discount certificates if the absorbing barrier and the bonus level are chosen adequately. However, because of these two arbitrary parameters, they are more difficult to handle than discount certificates, and calculating the gain and loss probabilities at expiration date becomes all the more important for the purchase decision.

Strengths of bonus certificates:

- If the absorbing barrier and the bonus level are chosen appropriately, it is possible to gain positive returns in flat or slightly decreasing markets.
- If the underlying asset rises beyond the bonus level, one can fully participate in these profits.
- By combining the absorbing barrier A and the bonus level B in the correct way, a broad spectrum of investment strategies can be realized. Each parameter pair (A, B) leads to a return distribution function that is similar or equal to the distribution function of a fixed term deposit, or a bet with varying odds, or the underlying asset itself.

Weaknesses of bonus certificates:

- In very weak markets or crash situations the buck is always passed to you, even if you had chosen a conservative variant of bonus certificate. If the underlying asset touches or breaks through the absorbing barrier during the certificate's lifetime, the bonus return is irrevocably lost, even if the price of the underlying asset at expiration time lies above the barrier.
- Because of the premium (difference between purchase price of the certificate and price of the underlying asset) that has to be paid in most cases, once the absorbing barrier has been crossed you lose more than if you had invested in the underlying asset itself.

A **theoretical description of bonus certificates** can be found in:
http://www.sigmadewe.com/fileadmin/user_upload/pdf-Dateien/Theorie_Bonuszertifikate.pdf (in German)

Bonus Certificates

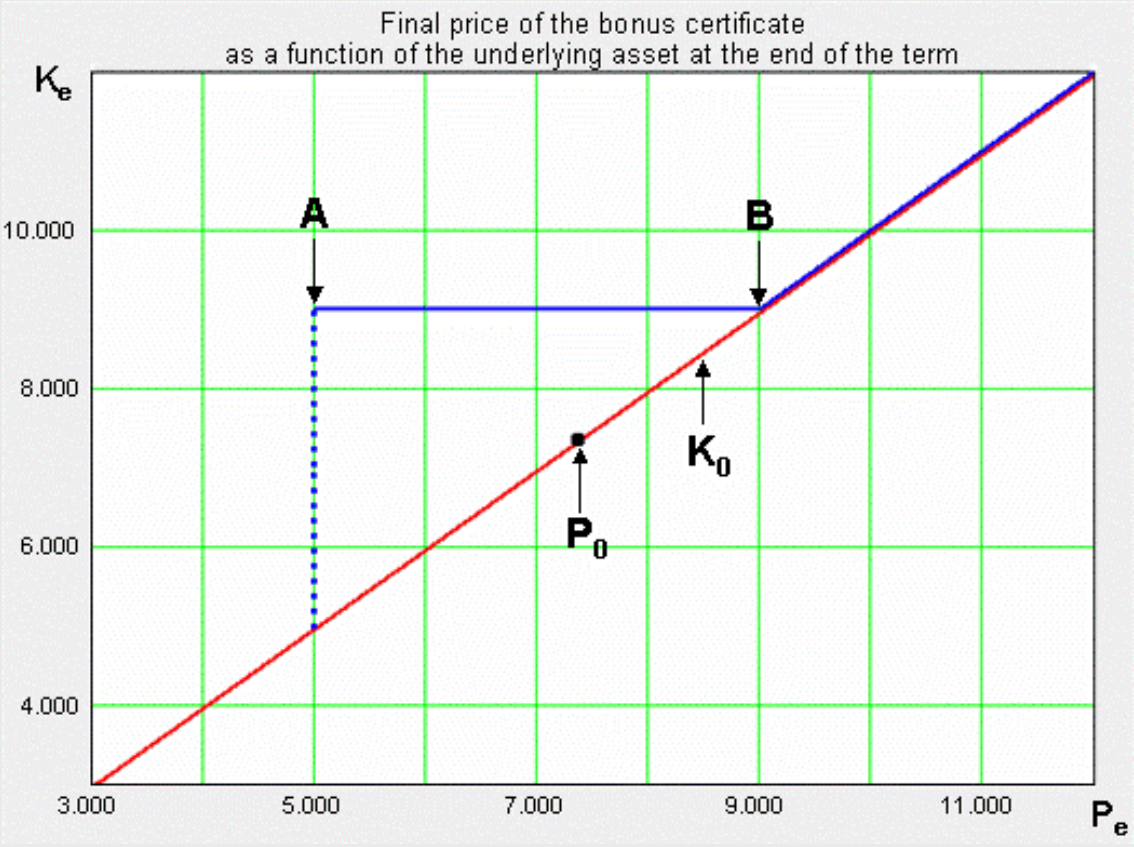
1. Design of bonus certificates

A bonus certificate is determined by two parameters, the absorbing barrier A and the bonus level B. The price of the bonus certificate at expiration time can be read off from the graph on the right. In this real example, the underlying asset is the German DAX index with a price P_0=7371 at purchase time. The chosen parameters for the absorbing barrier and the bonus level are: A = 5000, B = 9000. The price of the bonus certificate is K_0= 8453 (taking into account the subscription ratio). Lifetime of the certificate is 400 days.

Blue curve: final price K_e of the bonus certificate if the underlying asset has **never** touched or crossed the barrier during the certificate's lifetime.

Red curve: final price K_e of the bonus certificate if the underlying asset has touched or crossed the barrier at least once during The certificate's lifetime. In this case K_e becomes the final price P_e of the underlying asset and thus always lies on the red curve.

In contrast to discount certificates, the return of the bonus certificate at expiration time, $R_B = K_e/K_0 - 1$, not only depends on the final price P_e of the underlying asset but on its entire price history between issue and expiration time. For the case K_e = B the investor gets the bonus return $B/K_0 - 1$, in this example 6.5%.



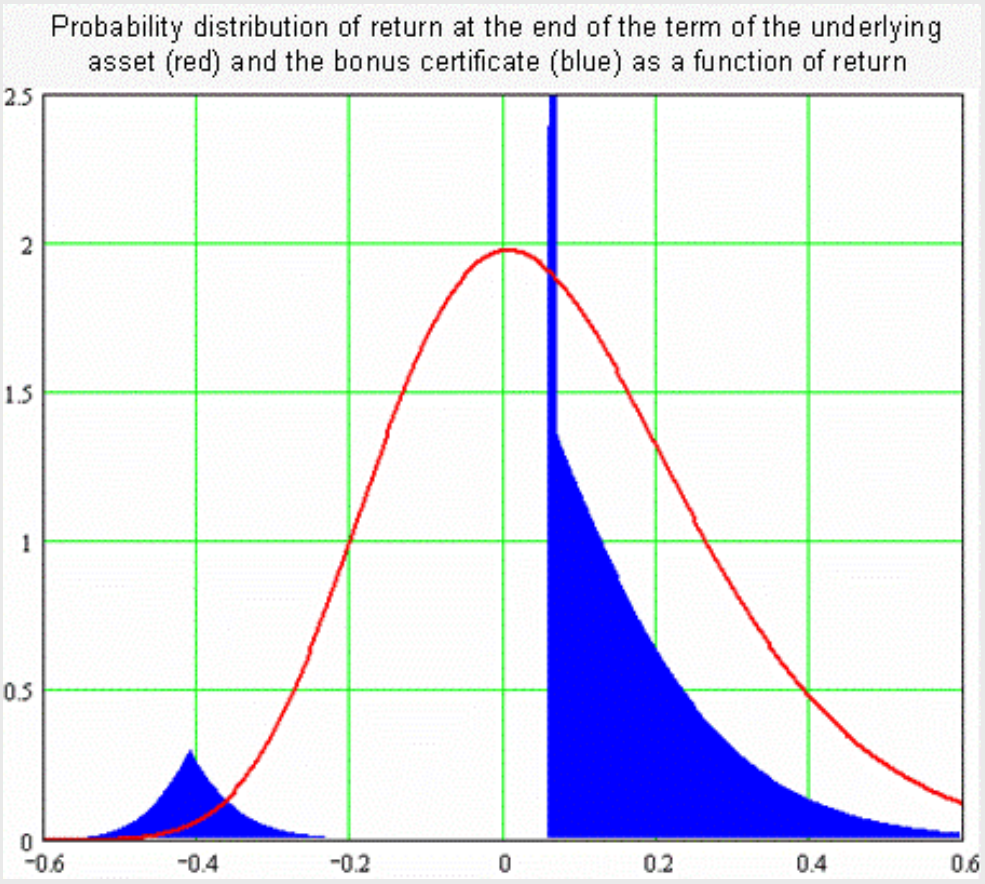
Bonus Certificates

2. Gain and loss probabilities

The differences between discount and bonus certificates are most obvious in the probability distributions of return at expiration time. While discount certificates usually have one peak at the cap, bonus certificates form two peaks, one at the bonus level and one at the absorbing barrier or below, if a premium was paid. This can be demonstrated with the real example from above.

Blue curve: probability distribution of return of the bonus certificate at expiration time. For this pair of parameters – absorbing barrier A and bonus level B - the characteristic split of the bonus certificate distribution function into two distinct peaks can be seen.

Red curve: probability distribution of return of the underlying asset at expiration time. This curve is generated by applying the Black Scholes model to the DAX index with assumed parameters: average annual return = 6%, annual volatility = 20%.



Bonus Certificates

2. Gain and loss probabilities

Results:

- In our example, the bonus certificate behaves like a typical betting situation: either one gains at least the bonus return of 6.5% (with the center of return at 9.4%) with probability p_g = 97%, or one suffers a heavy loss with a small probability p_l = 3%. In this case the maximum of the loss peak is at -40%. If one had invested directly in the underlying asset, the loss at the absorbing barrier would have been „only“ -32%.
- The fair price of this bonus certificate is K_f=8726, assuming a risk-free annual return of 2% and taking the subscription ratio into account. Thus the purchase price K_0 = 8453, fixed by the issuer, is below the fair price which is calculated with the parameters above for the underlying asset. As the fair price depends much more strongly on the volatility than on the average annual return of the underlying asset or the risk-free return, one can deduce that the issuer estimates the annual volatility at 28%.
- The correlation with the underlying asset is 0.72, i.e. the dependence on the underlying asset is still relatively high, but the product already clearly differs from a direct investment in the underlying asset. A correlation coefficient of 1 means that one could invest in the underlying asset directly. This case occurs for some parameter combinations, or whenever the barrier has been touched.
- The average annual return of this bonus certificate is 7.9%. Because of the bimodal return distribution function, this is less meaningful than the same value in discount certificates.

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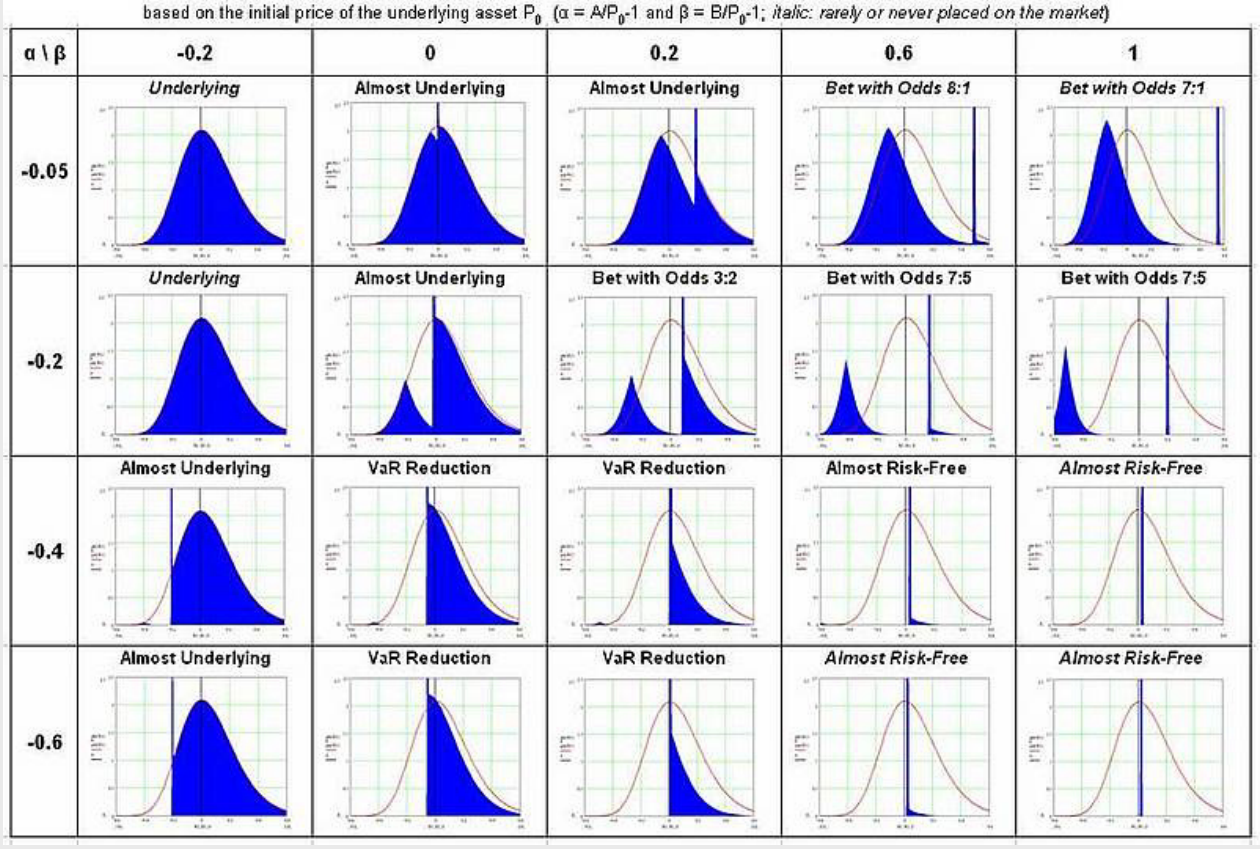
3. Parameter matrix

By means of a parameter matrix we demonstrate the dependence of the return distribution function of a bonus certificate on the choice of the two parameters, absorbing barrier and bonus level. The fictitious underlying asset does not distribute dividends and has the parameters: average annual return = 6%, annual volatility = 20%. The lifetime of the bonus certificate is one year. The risk-free return is assumed to be 2%.

The parameter matrix to the right shows how the nature of the bonus certificate changes when barrier A and bonus level B are varied. $\alpha = A/P_0 - 1$ in the first column and $\beta = B/P_0 - 1$ in the first row are, on a percentage basis, the distance of the absorbing barrier and bonus level, respectively, from P_0, the price of the underlying asset at purchase time. A table of all statistical values and fair prices can be found in **Theory of Bonus Certificates** (in German).

Probability distributions of return after one year of bonus certificates as a function of absorbing barrier A and bonus level B based on the initial price of the underlying asset P_0 ($\alpha = A/P_0-1$ and $\beta = B/P_0-1$; *italic: rarely or never placed on the market*)



Bonus Certificates

3. Parameter matrix

Results:

- **Underlying asset-like behavior:** The closer A is to P_0 at moderate bonus levels, the more similar the bonus certificate becomes to its underlying asset, because the barrier will be touched with high probability, which causes the bonus certificate to behave like the underlying asset. The "almost underlying“ asset behavior also results from choosing a bonus level below P_0, because the certificate behaves like the underlying asset whenever the price of the underlying asset lies above the bonus level. In these cases the correlation between certificate and underlying asset is close to 1. One gets exactly the underlying asset for $B \leq A$, a parameter combination that is not, however, offered in practice.
- **Bets with varying odds:** With bonus levels clearly above P_0 and barriers below P_0 by not more than about -35% the distribution function splits into two separate peaks. This is similar to a bet with varying odds between the buyer of the bonus certificate and the issuer. A good measure that defines this situation is the Value at Risk (VaR) which becomes larger than the VaR of the underlying asset.
- **VaR-reduction:** For absorbing barriers <-40% below P_0 in combination with bonus levels only slightly above P_0 the Value at Risk (VaR) can be reduced significantly with respect to the underlying asset. In the example with $\alpha = -0.4$ and $\beta = 0$, the VaR is reduced from 0.32 of the underlying asset to 0.05 of the bonus certificate, leaving out the worst 1% of all results.
- **Term deposit-like behavior:** This type of "almost risk-free" investment which is characterized by correlation coefficients of 0.3 and less, can be simulated by choosing very low lying absorbing barriers (-50% and more) and high bonus levels. But even for these so-called “deep bonus certificates” the buck is always passed to the buyer of the bonus certificate, if the underlying asset touches or breaks through the absorbing barrier if there is a market crash.

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4. Capped bonus certificates

A sub-class of bonus certificates is the capped bonus certificates. Here the bonus level assumes the additional role of a cap C which limits profits in the same way as the cap in discount certificates. The return distribution functions of capped bonus certificates show clear differences that are demonstrated by the parameter matrix to the right. In particular, the situation "almost risk-free" investment occurs more often than in classical bonus certificates. $\alpha = A/P_0 - 1$ in the first column and $\gamma = C/P_0 - 1$ in the first row are again, on a percentage basis, the distance of the barrier and the cap (=bonus level), respectively, from P_0, the price of the underlying asset at purchase time. The fictitious underlying asset has the same properties as in the parameter matrix for bonus certificates.

Probability distributions of return of capped bonus certificates after one year as a function of absorbing barrier A and cap C based on the initial price of the underlying asset P_0 ($\alpha = A/P_0-1$ and $\gamma = C/P_0-1$; *italic: rarely or never placed on the market*)

α \ γ	-0.2	0	0.2	0.6	1
-0.05	*Discount Certificate*	*Almost Discount C.*	Almost Discount C.	Bet with Odds 5:1	*Bet with Odds 5:1*
-0.2	*Discount Certificate*	Bet with Odds 3:2	Bet with Odds 7:5	Bet with Odds 7:5	Bet with Odds 7:5
-0.4	Almost Risk-Free	Almost Risk-Free	Almost Risk-Free	Almost Risk-Free	*Almost Risk-Free*
-0.6	Almost Risk-Free	Almost Risk-Free	Almost Risk-Free	*Almost Risk-Free*	*Almost Risk-Free*

Bonus Certificates

4. Capped bonus certificates

Results:

- **Discount certificate-like behavior:** For absorbing barriers A and caps C close to P_0 the capped bonus certificate resembles a discount certificate. For the theoretical case $C \leq A$ it is identical to a discount certificate. However, these parameter combinations are rarely offered in practice.
- **Bets with varying odds:** Typical bets can be realized with caps $C \geq P_0$ and barriers about -10% to -30% below P_0. For these typo parameter combinations the return distribution function splits into the two separate peaks that are characteristic for bets.
- **Term deposit-like behavior:** Because of the limitation on profits a capped bonus certificate simulates a term-deposit-like investment in many more cases than a bonus certificates with identical parameter combinations. These cases occur for absorbing barriers < -40%, independent of the value of the cap. As in the case of classical bonus certificates, one has to note that the buyer will suffer high losses whenever the underlying asset touches or breaks through the barrier during the certificate's lifetime.