Bayes and GAUs *

Probability statements about future major accidents at nuclear power plants after Fukushima, Chernobyl, Three Mile Island

Ulrich Leuthäusser

(01.04.2011)

*: GAU (= **G**rösster **A**nzunehmender **U**nfall) is the German abbreviation for the worst case scenario in a nuclear power plant. On the International Nuclear and Radiological Event Scale (INES) a "Super-GAU" has level 6 or level 7. In the following we are dealing with Super-GAUs.

Introduction

The aim of this short paper is to estimate the probability for the next Super-GAU (major accident, meltdown) in nuclear power plants (NPPs) using Bayes' theorem. The result is a simple formula by which one can also explain the different opinions about nuclear power plants.

The Bayesian approach starts with the definition of an a priori probability P(H) for the set of possible hypotheses H, before data are available. After data D have been observed, one can calculate the a posteriori distribution of H:

$$P(H \mid D) = \frac{P(H) \cdot P(D \mid H)}{P(D)}$$

In our case, H describes the accident rate in NPPs. The data D are both the number of accident-free years and the number of accidents that took place. From a priori knowledge and a priori assumptions, together with new information by the observations, more accurate knowledge about H arises.

Calculation of the probability of a major accident in the next few years

Let p be the probability of a Super-GAU (hereinafter referred to only as GAU) per year for a NPP. The a priori distribution of p is assumed to be

$$f_0(p) = (1-p)^m (m+1)$$

The expected value of p is $\int_0^1 f_0(p)p\,dp = \frac{1}{m+2}$. m is therefore the a priori average number of accident-free years of operation. Assuming m = 2.5 10^4 means that only every 25,000 years a GAU occurs in a NPP.

The probability of a probability p seems strange, but it is quite common in the context of a subjective probability conception, if p is not exactly known. The choice of the a priori distribution f_0 is a sensitive issue in the Bayesian method. In the present case, we choose a (special) Beta distribution for f_0 for three reasons: 1. p lies within the proper limits, 2. f_0 is conjugate (i.e. it belongs to the same family as the resulting a posteriori distribution), and 3. f_0 as a function of m offers a broad spectrum of distributions including the uniform distribution for m = 0.

The probability for the event $E_0(i)$ = "no accident within n years at the NPP i" is given by

$$P(E_0(i) \mid p) = (1-p)^n .$$

The probability for the event $E_1(i)$ = "1 accident at the NPP i in the nth year after n-1 accident-free years" is given by

$$P(E_1(i) \mid p) = p(1-p)^{n-1}$$

As can be seen in a moment, we formulate P explicitly as a conditional distribution. If N-1 NPPs had no accident during n years and one NPP has an accident in the nth year after n-1 accident-free years, then the overall probability is

$$P(E_0(1), E_0(2), \ldots E_0(N-1), E_1(N) \mid p) = p(1-p)^{Nn-1} .$$

Here we have assumed conditional independence between the NPPs which means

$$P(E_0(1), E_0(2), \ldots E_0(N-1), E_1(N) \mid p) = P(E_0(1) \mid p) \cdot P(E_0(2) \mid p) \ldots \cdot P(E_1(N) \mid p)$$

Using Bayes' theorem one can easily determine the a posteriori distribution:

$$P(p \mid E_0(1), E_0(2), \ldots E_0(N-1), E_1(N)) = \frac{P(E_0(1) \mid p) \cdot P(E_0(2) \mid p) \ldots \cdot P(E_1(N) \mid p) f_0(p)}{\int_0^1 P(E_0(1) \mid p) \cdot P(E_0(2) \mid p) \ldots \cdot P(E_1(N) \mid p) f_0(p) dp} =$$

$$= P(p \mid 1 \text{ GAU after } n \text{ years}) = \frac{p(1-p)^{m+Nn-1}(m+1)}{(m+1)\int_0^1 p(1-p)^{m+Nn-1} dp} = p(1-p)^{m+Nn-1}(Nn+m+1)(m+Nn)$$

The general case "x GAUs within n years" is approximately given by

$$P(p \mid x \text{ GAUs within } n \text{ years}) \cong \frac{(m+Nn+1)!}{x!\,(m+Nn-x)!} p^x (1-p)^{m+Nn-x} = Bt(x+1, m+Nn-x+1)$$

with Bt(a,b) being the beta distribution.

In the last equation we have neglected the effect that after a meltdown there are no further years of operation for this special NPP which is a good approximation for Nn>>1.
In reality, m varies and depends on the type of nuclear power plant, the geographic location, etc. This could be described with an additional distribution for m that averages Bt. Aging processes of NPPs are also not included here in order to keep the description as simple as possible.

With the above distribution we can now calculate the probability $P(k \mid x)$ that in the next k years at least one accident will happen, knowing that x accidents have occurred in the last n years. One gets:

$$P(k \mid x) = 1 - \int_0^1 (1-p)^{kN} P(p \mid x \text{ GAUs within } n \text{ years})\, dp = 1 - \frac{\prod_{i=0}^{x} (Nn+m+1-i)}{\prod_{i=0}^{x} (Nk+Nn+m+1-i)}$$

Because x << Nn+m, it follows

$$\boxed{P(k \mid x) \cong 1 - \left(\frac{Nn+m}{Nk+Nn+m}\right)^{x+1}}$$

This is a very simple formula which depends only on the a priori number m of accident-free years of operation of one NPP, the actual accident-free operating time of all NPPs N n, the future accident-free operating time N k, and the number of accidents x that already occurred. Of all the variables only m is not exactly known. As expected, it is simply added to the NPPs' accident-free years of operation N n.

Discussion

Proponents of nuclear power plants will assume a very large value for m, since then P(k|0) goes to 0 and therefore the number of accidents x plays no role (a large value of m means a great confidence in the reliability of NPPs). For this group, one accident virtually makes no difference because dP/dx goes to 0 for large m.

People who assess a higher risk to NPPs will choose a smaller value for m. The smaller m is the more importance is attributed to known accidents. For this group, the world is changing more whenever a new accident occurs.

In the figure below different P(k|x) are plotted as a function of future years k. The following values are chosen: $m = 2.5 \cdot 10^4$, n = 40 (accident-free years per NPP) and N = 442 (currently operating NPPs worldwide). Even if no accident had occurred so far (x = 0), the probability P(20|0) is close to 20% that in the next 20 years a major accident will occur.

This is surprising because one has chosen such a small value for the so-called "residual risk" 1/m. But the large number of 442 reactors substantially increases this "residual risk". The result also shows that the a priori assumption is consistent with reality, i.e. m has the correct order of magnitude. m too large with a P<<1 can therefore be excluded because it does not match the already occurred accidents.

After the three Super-GAUs at Fukushima, Chernobyl and Three Mile Island, the probability that in the next 20 years a similar accident will happen again has grown to over 50%. May everyone draw his own conclusions from this.

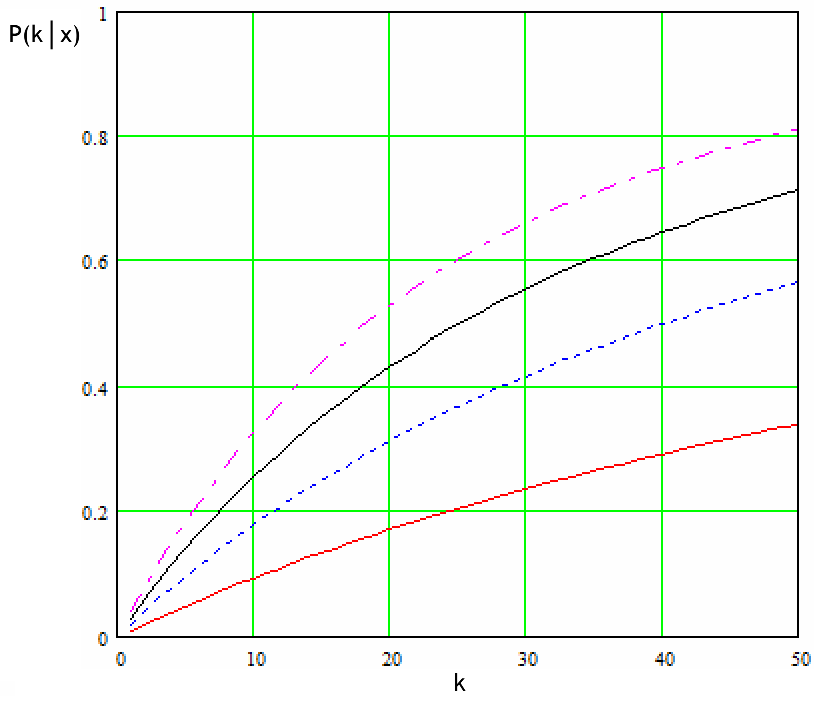

Abb.: Probability P(k|x) for a major accident in the next k years with the above assumptions for m, n and N, if in the past there had been no accident (red curve, x=0), 1 accident (blue curve, x=1), 2 accidents (black curve), 3 accidents (purple curve)