The dynamic failure process of a fiber bundle: an explanation of the fracture of a climbing rope

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This paper describes the fracture process of a fiber bundle model in a dynamic loading situation. First, a general system of equations for the resulting forces and elongations together with a fracture criterion is established. After presenting several exact solvable cases for different fiber breaking probabilities, a nonlinear model with a mixture of different fiber types is applied to the fracture process of a climbing rope. Good agreement with norm fall experiments is achieved.

1. Introduction

In physics and engineering, fiber bundle models (FBM) are important models to describe damage and fracture processes in disordered materials. These models already have a long history starting with the early work of Peirce [1] in 1926 who investigated the fracture behaviour of cotton yarns. An important milestone came from Daniels [2] with his statistical theory of bundles of threads. Because of their complex behaviour, FBMs are up to now important models for the description of material failure ranging from earth sciences, polymer physics, textile fiber mechanics and even social sciences. It is not surprising that the FMB is also a candidate to explain the fracture of a climbing rope: it has a parallel fiber structure, despite of its complicated kernmantle construction, where the core is made out of a number of cords consisting of several twisted strands which finally contain the single threads.

We follow the ideas of Daniels and describe in his words the fiber bundle (FB) as a collection of independent parallel threads so that all threads extend equally under tension: Starting from N_0 threads, the tension is equally distributed between the N_0 threads. If the load is large enough, some threads break and the load is redistributed among the remaining threads. Either a point is reached where the remaining threads have sufficient strength to maintain the load, or no such point is reached and all the threads give way, in which case the bundle is broken. Daniels understood the fracture process as a probabilistic problem. As a consequence, the breaking loads are statistically distributed and thus vary. This means for our application that a rope tested by a sequence of identical heavy falls does not break after always the same number of falls.

The paper starts in section 2 with the discussion of the dynamic failure mechanism of the FBM. Usually a static force is considered, but here we have a dynamic situation in which the FB has to hold a mass with a large initial velocity. A general system of difference equations is established for maximum elongations x_i, maximum forces F_i and surviving fibers N_i depending on the sequence of falls.
Because only little previous work on this topic exists in literature, our focus is first of all on a transparent discussion for some exact solutions of these difference equations for some simple but nontrivial models. Gradually advancing to more realistic models, the linear and the nonlinear case are discussed as well as different probability distributions with and without a cut-off for the breaking strength. In section 4 we also introduce a model for a bundle with a mixture of two different types of fibers. In section 5 the models of the previous sections are applied to a climbing rope, whose properties are shortly summarized at the beginning of that section. The intention is the explanation of the behaviour of the F_i, x_i and N_i and the number of falls to fracture i_{max} during the sequence of UIAA test falls.
A remark at the end about additional literature. There are some reviews about FBMs for physicists (see e.g.[3]), but little is found from the engineer's point of view [4]. The topic is not trivial and I have tried to make this paper as accessible as possible, but probably it is more appropriate for physicists interested in climbing than for climbers interested in physics.

2. The model and the fracture mechanism

Consider a rope made out of a bundle of N parallel fibers which have the same stress-strain curve f(x) not necessarily linear. The fibers have different breaking threshold values which are statistically distributed. Without such a distribution the bundle would either break immediately in the first fall or never. The distribution represents the disorder in the bundle and can arise from manufacturing differences and flaws. But even without that macromolecular disorder, where all fibers would break at the same elongation, heterogeneity can arise in form of a topological disorder. If the positions of the fibers in the bundle are not equivalent (e.g. because of helical structures) with fluctuating local elongations around the macroscopic extension, one also gets a failure distribution.

Let x denote the macroscopic strain variable and Q(x) = 1-P(x) the probability that a fiber breaks at x. As a distribution function for the elongation at fracture, Q(x) is nondecreasing and should not be confused with its probability density. An example is the widely used Weibull distribution. After breaking of NQ(x) fibers, the force is immediately and equally redistributed among the remaining intact NP(x) fibers and given by F(x) = NP(x)f(x). Note that F(x) is an expectation value describing average behaviour and is well defined only for large N (10^6 -10^7 for a rope). For small bundles, fluctuations of F should be taken into account.

Now the following dynamic situation is considered in which the fiber bundle is connected with a mass m which is accelerated by g over a falling distance and therefore has a certain velocity v_0 resp. kinetic energy E_0 when elongation starts. The restoring force F(x) slows down the mass and for large enough E_0 several of the weaker fibers fail so that a new fiber distribution arises. The next experiment is performed with the same initial velocity but with the changed new fiber distribution with less intact fibers and an increased force per fiber. This again leads to new fractures, and so on. Finally, after a sequence of i_{max} falls the fiber bundle breaks.
The physical starting point is the equation of motion $m\ddot{x} + Nf(x)P(x) + NP(x)\kappa(\dot{x}) = mg$ with given v_0 and x_0 = 0 and the velocity dependent friction term κ. Instead of treating that equation numerically, a more analytic description is chosen. Either applying energy conservation directly or multiplying the equation of motion by $\dot{x}$ and integrating it to the maximal elongation x_{max}, where the velocity is zero, one obtains

$$E_{kin}(0) = N \int_0^{x_{max}} f(x)P(x)dx + E_{diss} \tag{1}$$

g has been omitted in (1). This is possible for kinetic energies $E_{kin}(0) = m/2\, v_0^2$ much larger than the potential energy mgx_{max}. It was shown in [5] that for the Maxwell model the dissipated energy E_{diss} at maximum elongation is almost exactly proportional to the initial energy, i.e. $E_{diss} = \lambda E_{kin}(0)$. Energy dissipation therefore leads only to a rescaling of the initial energy with λ ~ 0.4. Instead of $E_{kin}(0)$ the scaled $E_0 = E_{kin}(0)(1-\lambda)$ is used in the following.

The sequence of fall experiments begins with the first fall experiment leading to $x_1^{max} \equiv x_1$. Repeating equation (1) one has

$$E_0 = N_0 \int_0^{x_1} f(x)P_1(x)dx = N_0 V(x_1) \tag{2a}$$

With the remaining fibers $N_1 = N_0 P_1(x_1)$, the equation describing the second fall takes the form

$$E_0 = N_1 \int_0^{x_2} f(x) P(x \mid x_1) dx = N_1 \int_0^{x_2} f(x) P_2^s(x) dx = N_1 V(x_2) \qquad (2b)$$

The fiber survival probability has changed from P_1 to $P_2(x \mid x_1) \equiv P_2(x)$. For the $(i+1)^{th}$ fall we have

$$E_0 = N_i \int_0^{x_{i+1}} f(x) P(x \mid x_1, x_2, \ldots x_i) dx = N_i \int_0^{x_{i+1}} f(x) P_{i+1}(x) dx = N_i V(x_{i+1}) \qquad (2c)$$

Note that in general P_{i+1} depends on all former falls.
Because of the decreasing N_i, equation (2c) may have no solution at all: the fiber bundle breaks for a certain $i = i_{max}+1$.

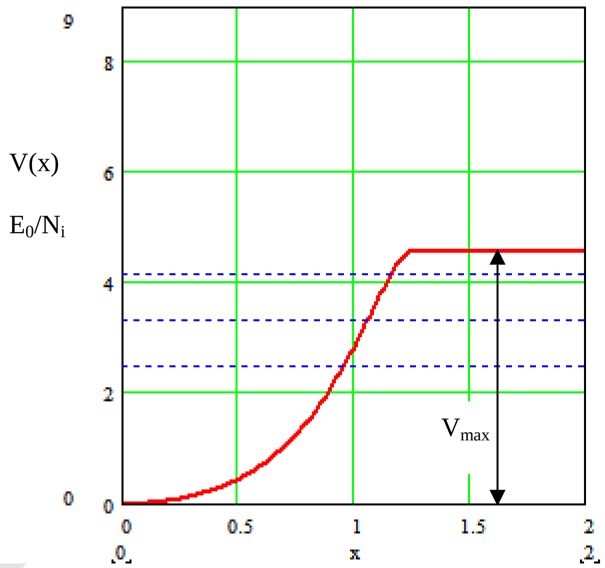


Figure 1. Illustration of the fracture mechanism. The red line is the potential V. If the blue lines representing the initial kinetic energy E_0 divided by the intact fibers N_i (see equations 2) are lower than V_{max}, no fracture occurs and each N_i (i = 1…3) leads to a new elongation x. V_{max} is analogous to a dissociation energy.

Solving formally equation (2c) for x_{i+1}, one obtains a system of difference equations for the breaking dynamics:

$$\begin{aligned} N_{i+1} &= N_i P_{i+1}\left[V_{i+1}^{-1}(E_0/N_i)\right] \\ x_{i+1} &= V_{i+1}^{-1}(E_0/N_i) \\ F_{i+1} &= N_i P_{i+1} f(x_{i+1}) = N_{i+1} f(x_{i+1}) \end{aligned} \qquad (3)$$

The failure of the complete rope takes place for a fall number i_{max} (in the sense of an average), if $x_{i+1} = V_{i+1}^{-1}(E_0/N_i)$ has no solution any more, i.e. $E_0/N_{i_{max}} > V(x)$ for all x.

In the next sections various probability models in combination of linear and nonlinear forces are presented.

3. Linear force and a survival probability P of Weibull type

The forthcoming models are two opposite approaches in order to understand the properties of the equations (3).

3.1 The same P(x) during all fall experiments without memory effects

Assuming that P(x) does not change during the falls, i.e. $P_{i+1}(x) = P_i(x)$, a fiber always breaks with the same probability at a certain strain x independent from the preceding sequence of experiments. This could be justified for the description of long falls with high energies.

We begin with a linear f(x) = kx and a probability $P(x) = e^{-bx^2}$ not to fail at strain x (as a special case of the general Weibull survival function e^{-bx^γ}). b is related to the reference length $1/\sqrt{b}$, the scale on which the fiber breaks. The maximum of $f(x)P(x)$ is $k/\sqrt{2be}$ at $\hat{x} = 1/\sqrt{2b}$. (2c) can be easily evaluated and one obtains

$$E_0 = N_i \int_0^{x_{i+1}} kxe^{-bx^2} dx = N_i \frac{k}{2b}\left(1 - e^{-bx_{i+1}^2}\right) = N_i \frac{k}{2b}\left(1 - \frac{N_{i+1}}{N_i}\right),$$

with the recursion relations for the surviving fibers

$$N_{i+1} = N_i - \frac{2bE_0}{k} \tag{4}$$

with a known initial N_0. Note that N_i is reduced by each fall no matter how small it is and thus cumulates the damage of all falls. From the $N_i = N_0 - (2bE_0/k) \cdot i$, the elongations and forces can be calculated

$$x_{i+1} = \sqrt{\frac{1}{b} \ln\left(\frac{N_i}{N_{i+1}}\right)}, \quad F_{i+1} = N_{i+1} \cdot kx_{i+1} \tag{5}$$

and are shown in the figure below.

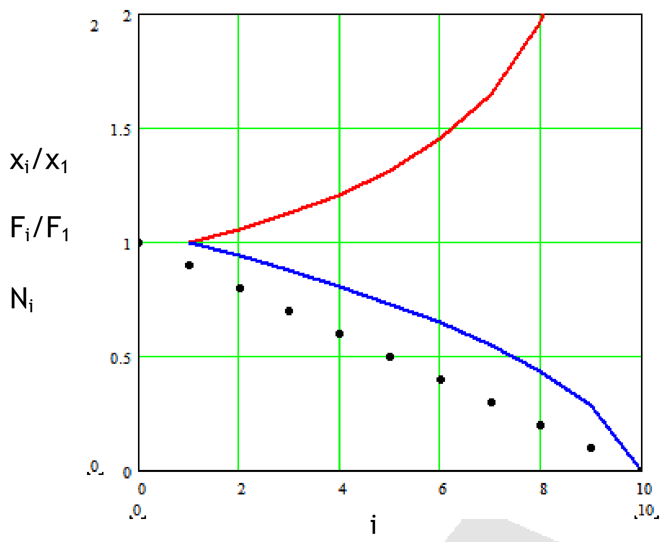


Figure 2. The relative maximum forces (blue), elongations (red) and surviving fiber numbers N_i (black) in dependence of the iteration number i. The force per fiber is proportional to x_i, thus producing more and more fractures.

The sequence i = 1, 2, … leads to increasing x_i and thus to increasing forces per fiber proportional to x_i inducing more and more fractures, whereas F_i and the P_i decrease.
The number of falls to failure is given

$$i_{max} = \left\lfloor \frac{N_0 k}{2bE_0} \right\rfloor \tag{6}$$

The bracket $\lfloor x \rfloor$ denotes the largest integer not greater than x. For this model a simple reciprocal relation between i_{max} and E_0 is obtained. Using the maximal modulus of toughness K_{max} given by $K_{max} = N_0 V_{max} = N_0\, k/2b$ equation (6) is simply the ratio between K_{max} and E_0.
Furthermore, we mention that the invariant P, i.e. $P_{i+1}(x) = P_i(x)$, can be easily generalized by the transformation $P_{i+1}(x) = \lambda \cdot P_i\,(x)^{\mu}$ allowing also exact results.

3.2 P(x) as a conditional probability including memory effects

Let us now discuss the opposite case where the stress history plays an important role. We take into account that some of the weaker fibers break in the sequence of experiments and therefore the fiber survival probability changes from fall to fall.
Suppose a fiber always survives a strain x_i if it survived it once, the conditional probability to survive an even higher strain x than x_i (i.e. the event x'>x given x'>x_i) is

$$\text{prob}(x' > x \mid x' > x_i) = 1 - \text{prob}(x' \le x \mid x' > x_i) = 1 - \frac{P(x_i) - P(x)}{P(x_i)} = \frac{P(x)}{P(x_i)} = P(x \mid x_i) = \frac{e^{-bx^2}}{e^{-bx_i^2}} \tag{7}$$

using the Weibull probability before. For $x \le x_i$ it is equal to one. The assumption that a fiber is resistant against strains which it already survived can be softened without difficulty introducing a constant aging parameter which replaces $P(x \mid x_i) = 1$ for $x \le x_i$. Because of

$N_{i+1} = N_i \cdot P(x_{i+1} \mid x_i) = N_i e^{-bx_{i+1}^2 + bx_i^2} = N_0 e^{-bx_{i+1}^2}$ one now has a weaker dependence on the fall number i (extending the lifetime of the rope) in contrast to the preceding model with $N_{i+1} = N_i e^{-bx_{i+1}^2}$ (see Fig.4).
Evaluating (2c) again, one obtains the difference equation

$$N_{i+1} = N_i\left(1 + \ln\left(\frac{N_0}{N_i}\right)\right) - \frac{2bE_0}{k} \qquad (8)$$

This is an interesting equation. Its continuous counterpart $\dot{N} = -N\ln\left(\frac{N}{N_0}\right) - \frac{2bE_0}{k}$ is similar to Gombertz equation which can be solved exactly, but the term $2bE_0/k$ in (8) prevents an exact solution. Equation (8) has a bifurcation point where its behaviour changes qualitatively: for energies $E_0/N_0 < k/2be$ there is a positive asymptotic value for $N_\infty > 0$, and the fiber bundle never breaks. Above that threshold, the corresponding relation for (6) is given by

$$i_{max} = \int_0^1 \frac{dx}{x\ln(x) + \frac{2bE_0}{kN_0}} \qquad (9)$$

only implicitly available. However, there is an excellent approximation $i_{max} \approx \left(\frac{k/2b}{E_0/N_0 - k/2be}\right)^{\frac{3}{4}}$. In the figures 3 and 4 below, a comparison between this model and the model from the last section is shown.

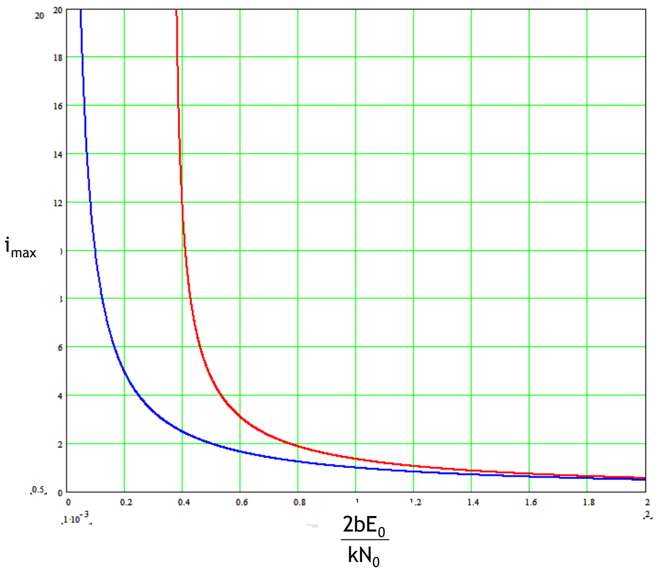


Figure 3. Plot of eq.(9) (red) and eq.(6) (blue). For the model underlying eq.(9) the bundle does not break below a threshold, whereas the model of eq.(6) fails always after a finite number of falls.

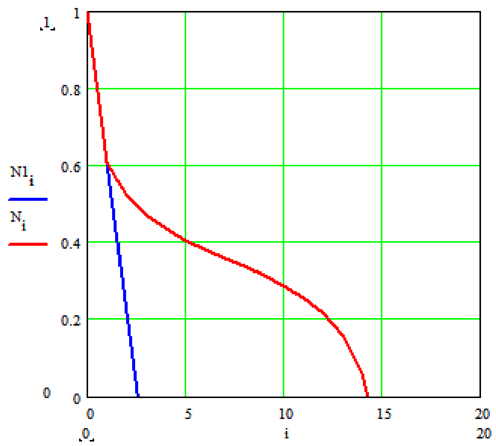


Figure 4. The survived fibers as a function of the iteration number i, shown for the model of the last section ($N1_i$, blue) and in red (N_i) the model of this section for a fixed $2bE_0/kN_0$ = 0.39, chosen near the threshold energy.

The maximum forces and elongations given by $x_{i+1} = \sqrt{\frac{1}{b}\ln(N_0/N_{i+1})}$ and $F_{i+1} = N_{i+1} \cdot kx_{i+1}$ behave similar to the first model (3.1) with increasing x_i and falling F_i.

4. Nonlinear forces

To get a more realistic description of a rope we have to extend our description to nonlinear forces. For small extensions, one should still have the linear elastic material treated above. Large extensions of the fiber, however, lead to a strong nonlinearity with a much stiffer behaviour which is also responsible for a larger energy storage capacity. The reason for this stiffening is that the fiber has a finite maximum length and that the deformations in a fall experiment under UIAA test conditions come close to it. Note that the survival probability itself is a source of nonlinearity in F(x) = NP(x)f(x) but acts in the opposite direction than the force coming from elastomer behaviour.

4.1 Piecewise constant survival probability

We have treated survival probabilities which decreased (section 3.1) with the number of test falls as well as an increase (section 3.2), because the weaker fibers gradually disappear. Now we discuss the case of a piecewise constant survival probability.
The survival probability in this section is equal to one up to x_α and is p = const up to x_β where it drops to zero (see Fig. 5 below).

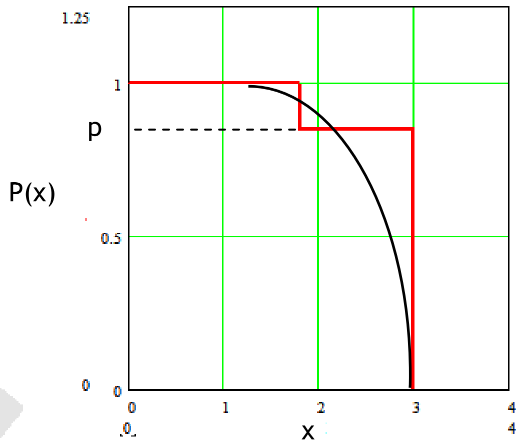


Figure 5. Schematic plot of the survival probability P(x) which can be viewed as an approximation of a more realistic smooth curve. Note that x_α =0 is possible leading to an even simpler P.

Because of the constant p, one has $N_{i+1} = p \cdot N_i$. Thus, one has without specifying f(x):

$$N_i = \begin{cases} N_0 & \text{if } x_1 < x_\alpha \\ N_0 p^i & \text{if } x_\alpha \le x_1 \\ 0 & \text{if } x_i \ge x_\beta \end{cases} \qquad (10)$$

x_α is the threshold which x_1 has to exceed in order to start the cascade of fiber failures with a corresponding threshold energy of $E_0^\alpha = N_0 \int_0^{x_\alpha} f(x)dx$. If $x_1 < x_\alpha$ no fiber breaks at all with the same N_0 for the next fall and so on. Thus, no fatigue and abrasion effects below

E_0^α are considered. This can be included without difficulty by an aging parameter q appearing at first for the second fall described by $E_0 = \left[N_1 \int_0^{x_\alpha} f(x)dx + N_1 p \int_\alpha^{x_2} f(x)dx \right] q$ and all following falls. Because of the cut-off x_β, the maximum potential is equal to the maximum static work capacity given by $V_{max} = \left(\int_0^{x_\alpha} f(x)dx + p \int_{x_\alpha}^{x_{max}} f(x)dx \right)$ and the fracture criterion takes the form

$$i_{max} = \left\lfloor 1 + \frac{\ln\left(\frac{N_0 V_{max}}{E_0}\right)}{\ln\left(\frac{1}{p}\right)} \right\rfloor \qquad \text{or} \qquad E_0 \left(\frac{1}{p}\right)^{i_{max}-1} = \text{const} \tag{11}$$

As in equation (6), i_{max} is again determined by $K_{max} = N_0 V_{max}$, the maximal modulus of toughness, divided by the initial kinetic energy but now in a logarithmic relation to i_{max}. For $E_0 < E_0^\alpha$, i_{max} goes to infinity.

Let us now specify the nonlinearity. For elastomers the physical description leads to a stress-strain relation given by the inverse Langevin function [6]. However, this complicated expression can be treated only numerically and moreover the underlying model of an elastomer consisting of chain molecules with free rotation about their joints cannot be taken too literally in our case with all its restrictions. Therefore we use a force $f(x) = kxe^{cx^2}$ which is similar to the Langevin function in a large region of the strain and which gives simple expressions. Calculation of (2c), i.e. $E_0 = N_i \left(\int_0^{x_\alpha} f(x)dx + p \int_{x_\alpha}^{x_{i+1}} f(x)dx \right)$ with $f(x) = kxe^{cx^2}$, yields $E_0 = N_0 p^{i-1} \frac{k}{2c}\left(e^{cx_\alpha^2} - 1\right) + N_0 p^i \frac{k}{2c}\left(e^{cx_i^2} - e^{cx_\alpha^2}\right)$. Solving for x_i, one obtains the elongations and forces:

$$x_i = \sqrt{\frac{1}{c} \ln\left(\frac{1}{p} - e^{cx_\alpha^2}\left(\frac{1}{p} - 1\right) + \frac{2cE_0}{kp^i} \right)} \tag{12}$$

$$F_i = N_0 p^i k x_i e^{cx_i^2} \tag{13}$$

The force for the last possible fall before fracture ($i = i_{max}$) becomes $F_{max} = N_0 p^{i_{max}} k x_\beta e^{cx_\beta^2} = \frac{E_0 p}{V_{max}} k x_\beta e^{cx_\beta^2}$, whereas the maximum static and dynamic breaking strength (first fall) is $N_0 p k x_\beta e^{cx_\beta^2}$, so that $F_{max} / F_{max}^{static} = E_0 / N_0 V_{max}$ which directly determines i_{max}. The breaking strength is linear in N_0 and therefore simply scales like the cross-sectional area of the rope. The maximum potential appearing in these expressions is given by $V_{max} = k \frac{1}{2c}\left(p(e^{cx_\beta^2} - e^{cx_\alpha^2}) + e^{cx_\alpha^2} - 1 \right)$. Comparing F_{max} with the force of the first fall

$F_1 = N_0 p k x_1 e^{c x_1^2}$, the ratio $\frac{F_{max}}{F_1} = \frac{N_0 p^{imax} k x_\beta e^{c x_\beta^2}}{N_0 p k x_1 e^{c x_1^2}} = \frac{E_0}{V_{max}} \frac{x_\beta}{x_1} e^{c(x_\beta^2 - x_1^2)}$ can now be much larger than one because of the nonlinearity parameter c.

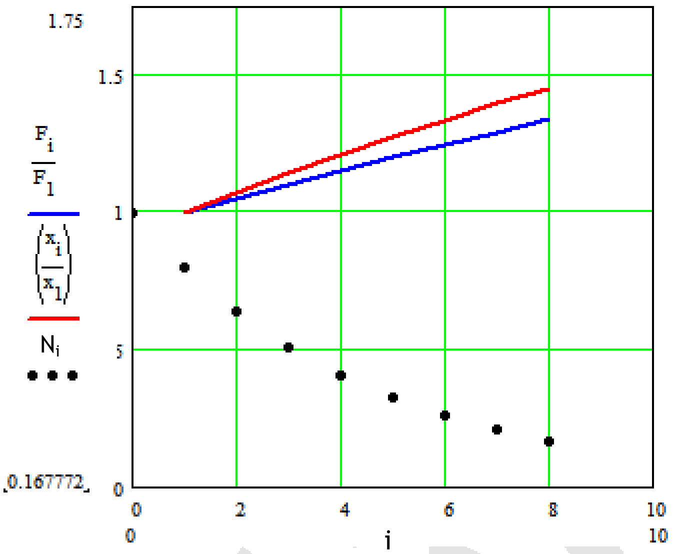


Figure 6. The relative forces (13) and elongations (12) together with the N_i (10) are plotted against fall iteration number i. The nonlinearity c is responsible for the increase of F_i.

4.2 Piecewise constant survival probability which varies during the fall sequence

The results from the last section can be generalized. As we have seen in section 3, the survival probabilities normally vary during the sequence of fall experiments. This can be incorporated in the model of 4.1. We have only to solve the slightly more complicated equation

$$E_0 = \left[N_{i-1} \int_0^{x_\alpha} f(x)dx + N_{i-1} p_i \int_{x_\alpha}^{x_i} f(x)dx \right] \quad (14)$$

As before, the fractures begin only if $x_1 \geq x_\alpha$ and the changes of Ni are again a proportion of its previous value, $N_{i+1} = p_{i+1} N_i$ and Ni is now given as a product of all former p_i

$$N_i = N_0 \prod_{j=1}^{i} p_j \quad (15)$$

The strains and stresses have the same form as (12) and (13):

$$x_i = \sqrt{\frac{1}{c} \ln\left(\frac{1}{p_i} - e^{c x_\alpha^2} \left(\frac{1}{p_i} - 1 \right) + \frac{2cE_0}{kN_i} \right)} \quad \text{or} \quad x_i = \sqrt{\frac{1}{c} \ln\left(\frac{N_{i-1}}{N_i} - e^{c x_\alpha^2} \left(\frac{N_{i-1}}{N_i} - 1 \right) + \frac{2cE_0}{kN_i} \right)} \quad (16)$$

$$F_i = N_i k x_i e^{c x_i^2} \quad (17)$$

We now show how a varying p can arise. Assume that there is a part Q of the bundle which survives with a higher constant probability $\overline{q}$ than the other part M with constant probability $\overline{p}$ (a kind of a composite fiber bundle). There are several possible reasons for the appearance of two or more different parts: except for a real mixture of different yarn types, the fibers of the outer part are more likely to fail than those of the more protected inner part. M could also represent localized areas where the cords of the rope kernel have strong contact with each other leading to more abrasion because of the larger friction. Local force differences are a further reason for heterogeneity.
The fiber survival probability p_i for the whole bundle is of the form

$$p_1 = \frac{N_1}{N_0} = \frac{Q_1 + M_1}{Q_0 + M_0} = \frac{Q_0\overline{q} + \overline{p}M_0}{Q_0 + M_0}, \quad p_2 = \frac{N_2}{N_1} = \frac{Q_2 + M_2}{Q_1 + M_1} = \frac{Q_0\overline{q}^2 + \overline{p}^2 M_0}{Q_0\overline{q} + \overline{p}M_0}, \ldots$$
$$p_i = \frac{N_i}{N_{i-1}} = \frac{Q_0\overline{q}^i + \overline{p}^i M_0}{Q_0\overline{q}^{i-1} + \overline{p}^{i-1} M_0} \qquad (18)$$

Because the fraction M dies out faster, the probability p_i for the whole bundle rises, although $\overline{q}$ and $\overline{p}$ of the parts are constant. For $Q_0 = 0$ or $M_0 = 0$ we find the old result of a constant p. The fracture criterion for varying p is more complicated than before and is discussed in the next section.

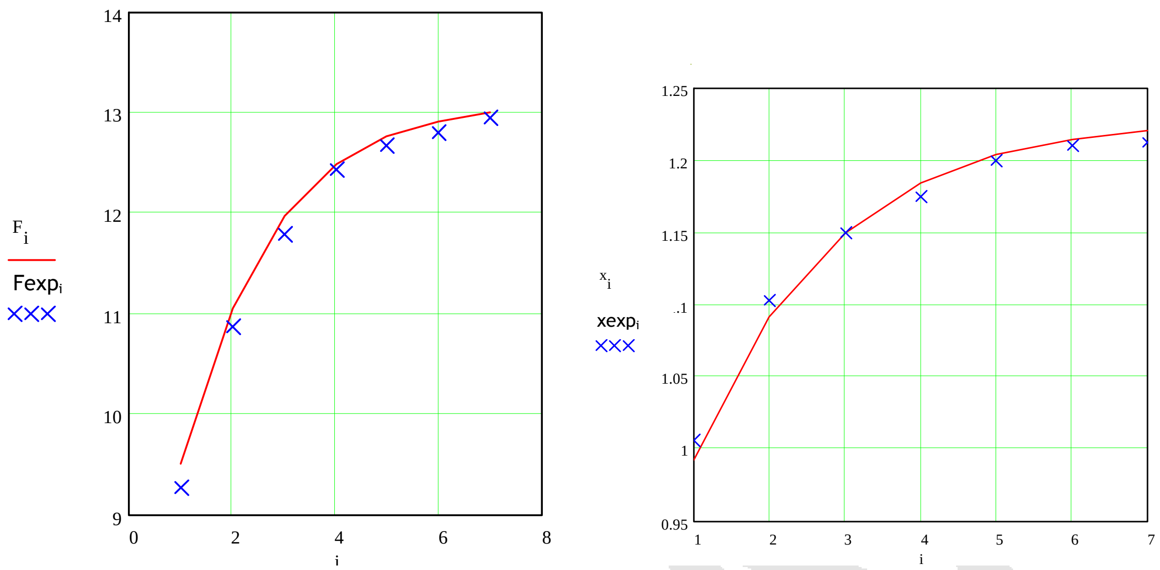
5. Application to a climbing rope

We are now ready to apply the previous results to a climbing rope using the model from the last section (4.2). First, some properties of a climbing rope are listed which a good model should fulfill

1. maximum impact force of the first fall* ranges from 7kN – 9.5kN (dependent on the diameter and the rope manufacturer)
2. maximum impact force of the last fall* before failure is about 45% higher than for the first fall [7,8], thus increases with the number of falls
3. wide range of maximum number of test falls* to fracture between 5-14
4. maximum dynamic strain of the first fall* is between 30%-37% or roughly 1m in absolute numbers
5. maximum dynamic strain of the last fall* is about 20% higher, thus increases with the number of falls [7,8]
6. static elongation (mass m = 80kg*) between 7% and 10%
7. static breaking strength is about 20 kN corresponding to a mass of 2 tons under gravity g with a maximum elongation of about 50%
8. dynamic breaking strength in the first fall* is estimated at about 4 times of the standard mass, i.e. 320kg
9. nonlinear stress-strain curve for larger stresses
10. elasticity modul is around 0.5 GPa calculated from linear viscoelastic theory [5]
11. energy absorption* until maximum elongation is about 40% [5]

(*under UIAA norm test conditions)

In the figures below, the maximal impact forces, the corresponding elongations together with the experimental data under UIAA test conditions are shown. These data were made available by the IFT of the University of Stuttgart [7]. Other measurements provided by Edelrid [8] are completely consistent with [7].



Figures 7a,b. Comparison between the experimental values (crosses) [7] and the calculations (solid lines) of the sequence of impact forces F[kN] and elongations x[m] under UIAA test conditions for a typical climbing rope.

We emphasize that for the good agreement between theory and experiment both a nonlinear force F and increasing survival probabilities p_i are important. The nonlinearity parameter c is about 1.5. The increase of F could in principle be explained by a change of the rope viscosity. This, however, would involve a longer time to reach the force maximum which is not observed [7]. M_0, the fast breaking part (18), is assumed to be about 2 times larger than the long living part.

In the next figure, the number of falls to failure is (logarithmically) plotted against the initial fall energy E_0 for the model of this section compared to equation (11) coming from the model with constant survival probabilities. The red curve is close to the experiments of Pavier [9], one of the few profound investigations for the failure analysis of climbing ropes.

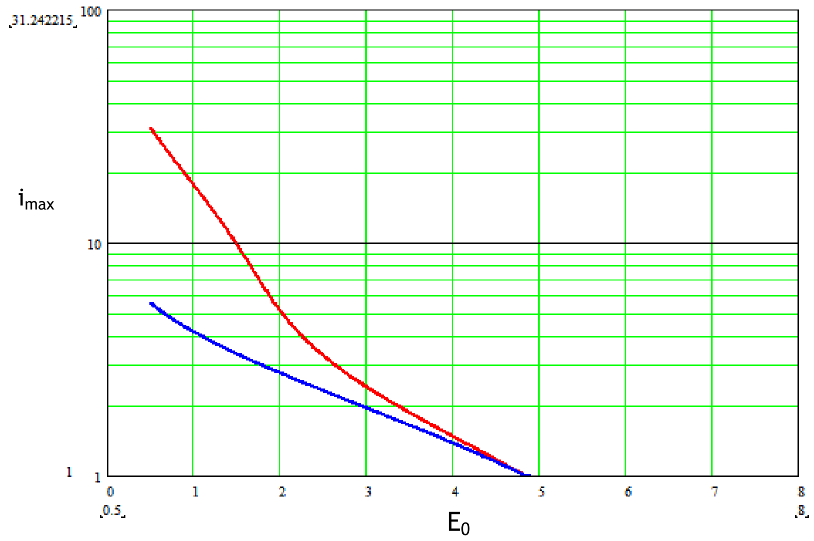


Figure 8. Maximum number of falls vs. initial energy E_0. The splitting into two parts with different fiber survival probabilities leads to drastic differences for small energies between the model from section 4.2 (red) and model from section 4.1(blue). The red model can bear much more falls with small energies E_0.

In the limit of small elongations, the presented breaking mechanism predicts a softening of the rope for growing fall numbers. The experiments are contradictory: the measurements of the IFT [7] show almost no change (i.e. linear elastic behaviour for small deformations is almost constant during the sequence of falls), whereas the measurements of [9] show softening as equation (17) predicts. It is known [11] that a broken fiber in a twisted structure still can carry small loads. Therefore it is possible to assume a spring constant $k>0$ also for the broken fibers for small elongations. For the breaking mechanism this assumption plays no role - it is only important that the broken fibers cannot carry load for large strains.

6. Conclusions

In this paper we analyzed the fracture mechanism of a bundle of a large number of parallel and independent fibers which have to hold an attached mass with a given fall energy.
If the energy is large enough, the fibers break with a certain probability, thus the FB accumulates damage (usually above a certain threshold) during the sequence of falls.
For every new fall, the maximum strain increases because of less intact fibers leading to a higher force per fiber which induces new failures in the next fall. In addition, a nonlinear force leads to a strong stiffening of the bundle resulting in an increased maximum impact force. Finally, the bundle breaks when its energy storage capacity is lower than the fall energy of the mass.
Using energy conservation including the energy absorption by internal friction, a system of recursive equations has been derived for the maximum forces, elongations and the surviving fibers as a function of the number of falls. A fracture criterion relates the number of falls to fracture to the initial fall energy.

The main interest was to apply the FBM and to explain the failure of climbing ropes. However, this analysis is not limited to this type of ropes, further applications are static ropes and cords, caving and canyoning ropes.
Despite of some simplifying assumptions, the model from section (4.2) is in good agreement with the experimental facts. Although there are several adjustable parameters, it is not trivial to get a quantitative agreement (see Fig.7) for forces and elongations in absolute numbers. The conclusion is that the key properties of the fracture process are captured by our approach and a refinement to more complicated models with numerical calculations would only be necessary for new more detailed experiments.

In order to reproduce the experiments, it was important to choose a strong nonlinear force. Furthermore, against intuition, the fiber survival probabilities increase with fall number. One would rather expect a decrease because of aging processes, but these processes are not relevant for the heavy falls of the standard UIAA tests. The increase can arise either from the extinction of weak fibers because of a broad breaking strength distribution or from a mixture of at least two different parts of fibers.

The many possibilities for the choice of P(x) and the richness of equations (3) leave open questions which could be answered by additional measurements. So it would be very easy to vary the mass or the initial fall velocity in the tests in order to validate some of the assumptions and to get more information about the failure distributions which are the key for a better understanding of the fracture process. This in turn could improve the performance of ropes by increasing their durability.

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