# Discount Certificates

Suppose you have identified what proportion of your portfolio you are willing to expose to risk and you are currently in the market because of positive market signals. However, you are not sure that the market will continue to grow in the medium term. So you want to achieve a better return than the risk-free rate even in a flat or slightly declining market. This is possible by investing in a discount certificate, especially in one whose cap lies below the current price of the underlying asset.

To profit optimally from the strengths of discount certificates, one has to understand their behavior as a function of the cap. The crucial step in modeling discount certificates is to determine the "fair price", which is obtained by setting the Sharpe ratios of the underlying assets and the discount certificates equal. Knowing the fair price, one can also determine what volatility the issuing banks are assuming when calculating the price of the certificate, or, looked at in a different way, one can find out if the bank's discount certificate price corresponds to a fair bet between bank and consumer. Our calculations will demonstrate the strengths and weaknesses of discount certificates:

**Strengths:**

- Fluctuations in the part of the portfolio that is exposed to risk can be reduced at the expense of only minor losses in the return.
- It is possible to gain positive returns in flat or slightly decreasing markets.

**Weaknesses:**

- In strongly rising markets the return is smaller than with a direct investment in the market index.
- In very weak markets, i.e. markets that fall below the buffer zone (the price gap between the price of the underlying and the price of the discount certificate) the losses are identical to the direct investment in the market index.

The **theory of discount certificates and their iteration** can be found in:

http://www.sigmadewe.com/fileadmin/user_upload/pdf-Dateien/Theorie_Discountzertifikate.pdf (in German)

# Discount Certificates

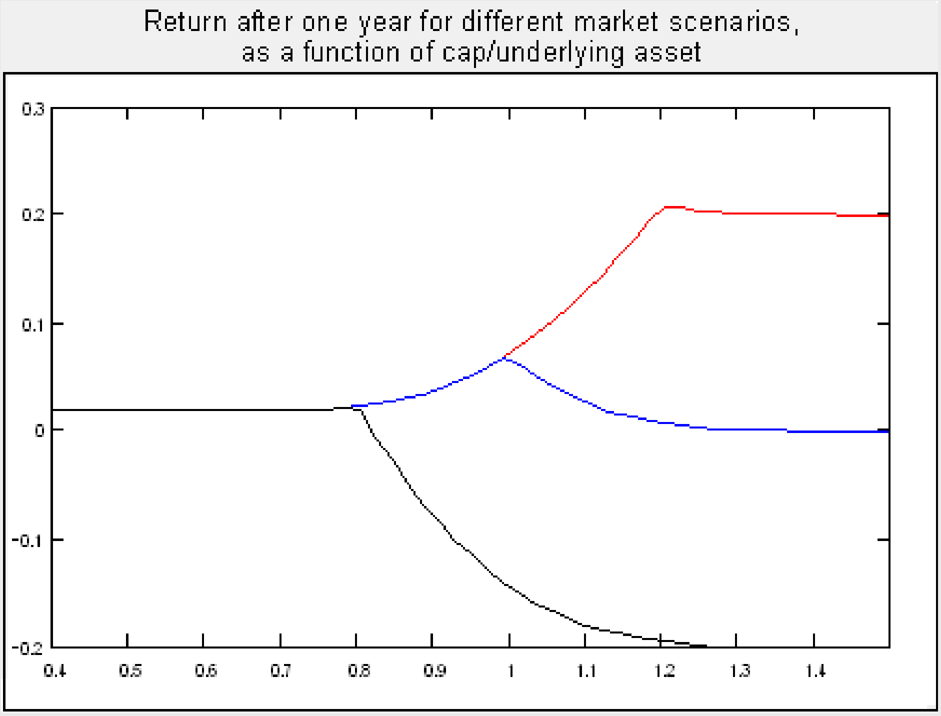
## 1. Scenario analysis

To get a feeling for how to choose the cap, we consider three different scenarios for the market development after the purchase of a discount certificate. In the first case, at expiration time of the discount certificate the price of the underlying asset is 20% below its current value. In the second case, the price of the underlying asset stays the same, and in the third case, the price of the underlying asset has increased by 20%. The term of the discount certificate is one year. The underlying asset is a market index, e.g. the German DAX index, with an assumed annual volatility of 15% and an average yield of 0% (flat market). The average risk-free return is 2%. The following graph shows the return of the discount certificate at expiration time as a function of c (c=cap/price of underlying asset at purchase time).

**Black curve:** return after one year if the market index has fallen by 20%.

**Blue curve:** return after one year if the market index has the same value as when purchased.

**Red curve:** return after one year if the market index has increased by 20% relative to its current value.



# Discount Certificates

## 1. Scenario analysis

**Results:**

- Discount certificates are most valuable when the market index at expiration time is the same as at purchase time. If you choose a cap close to the current market index price (c=1), you would achieve a return of 6.9%.
- If the market index had declined by 20% after one year and you had again chosen a cap close to the current market index, your loss of 14% would have been less than with a direct investment in the market index. However, if you had chosen a smaller cap, for instance a cap 20% below the current market index (c=0.8), you would have achieved a positive return of 2.4%.
- If the market index grows by 20%, you give away return whenever you were cautious and chose a a cap smaller than the market index. Only at a cap of 20% above the current market index (c=1.2) is it possible to participate fully in the market gain. The return of 20.8% is even slightly more than the comparable gain of the direct investment in the market index. However, if you were wrong about the market direction, a cap larger than the market index doesn't protect against loss: if the market index ends up 20% lower than the initial value, the loss would amount to 19.4% for c=1.2.

# Discount Certificates

## 2. Risk-return behavior as a function of cap and loss probability

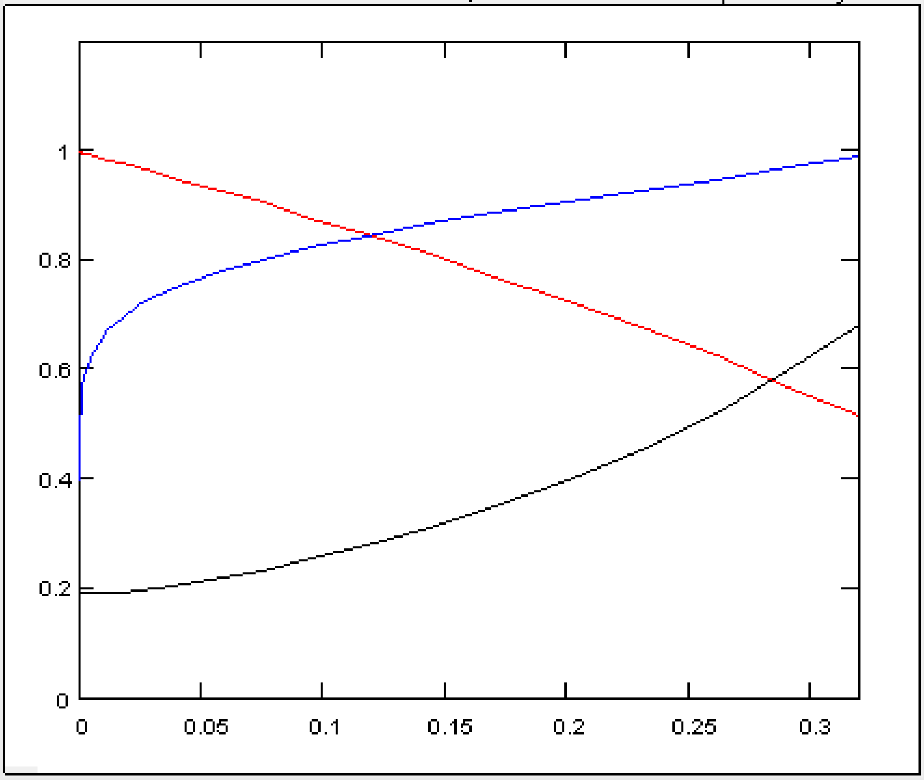
In this example we show for different loss probabilities how the maximum possible return of a discount certificate depends on the cap. The time period before expiration is again chosen to be one year, the risk-free return is 2%, market volatility is 15% and the assumed average yield on stock is 0% (flat market condition). This time we only consider discount certificates with caps up to the price of the underlying asset. The following graph shows three curves as a function of the loss probability:

**Black curve:** the maximum possible return $r_{max}$ after one year which is given by: $r_{max}$ = (cap - purchase price of the discount certificate) / purchase price of the discount certificate; $r_{max}$ is multiplied by a factor 10 in order to use the same scale of the y-axis for all three curves.

**Blue curve:** plot of c, where c = cap/underlying asset value, as a function of loss probability. Each value of c thus corresponds to a value of $r_{max}$ for a given loss probability.

**Red curve:** the probability to achieve $r_{max}$. For a given loss probability each value of $r_{max}$ thus corresponds to a value of c and a probability to achieve $r_{max}$.

Maximum possible return after one year, related cap, and the probability to achieve the maximum return, as function of loss probability



# Discount Certificates

## 2. Risk-return behavior as a function of cap and loss probability

**Results:**

- The maximum possible return $r_{max}$ achieves its maximum value of 6.9% for a cap that equals the price of the underlying asset (c=cap/underlying asset=1). Yet the 6.9% return is only achieved with a probability of 51.7%, the loss probability is 32.0%.
- In the opposite case c -> 0, i.e. the cap is much smaller than the price of the underlying asset, one can only achieve the 2% risk-free return, with a loss probability close to zero (this is an alternative to term deposits).
- If one had invested the same amount directly into the stock market, the most probable return would have been 0% at a loss probability of 50%.
- With deep discount certificates one can achieve a return higher than the risk-free return, with almost no risk. This is even more true since one can sell a discount certificate at any time before their expiration date (see **practical tips**).

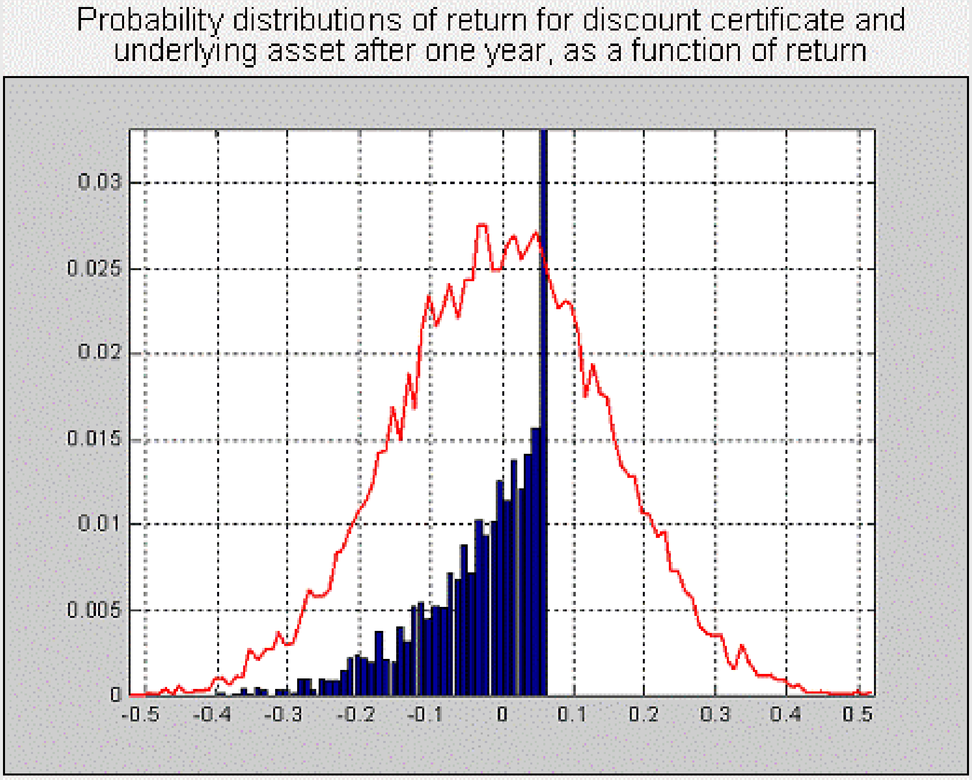
# Discount Certificates

## 3. Comparison with a direct investment in the market index

Here we compare the analytic results of the two preceding examples with the corresponding direct investment in the market index. We demonstrate how the two investments differ in returns, volatilities, and loss probabilities. To this end, we simulate the stock market and show the return distribution function for investments both in a discount certificate and directly in the market index. We choose a real DAX discount certificate available on the market with a low cap (cap = 5100; value of the underlying asset = 5813; cap/underlying asset = 0.88). The price of the certificate, $K_0$, is 4820 – a discount of 17.1% below the price of the underlying asset -, and the duration of the certificate is one year. As in the first example, we assume that the market is flat for one year (0% for the expected yield of the stock index) and has a volatility of 15%. The graph shows the possible returns of the discount certificate (blue curve) and the possible returns of the underlying asset (red curve). The discount certificate's returns are limited to $r_{max}$ = (cap – $K_0$)/$K_0$, but in return, losses are smaller than for the direct investment in the market index.

**Blue curve:** return distribution function of the discount certificate after one year.

**Red curve:** return distribution function of the underlying stock index after one year.



# Discount Certificates

## 3. Comparison with a direct investment in the market index

**Results:**

- The return distribution function of the discount certificate has its maximum exactly at $r_{max}$, with a return value of at most 5.8%. The simulation yields a probability of 80% that this maximum return will be achieved. The return distribution function of the market index has its maximum at 0%, which is the expected yield of the stock index and is a parameter of the simulation.
- The simulation yields a volatility of the discount certificate of only 5.6% in comparison with the 15.0% volatility of the market.
- The loss probability for the direct investment in the market is 50%, but only 11.6% for the investment in the discount certificate (result of the simulation).

# Discount Certificates

## 4. Iterations of discount certificates

Using certificates for long-term investment automatically requires "rolling", or replacing, the certificate after its expiration. For long time periods this iteration of certificates leads to return distribution functions which may differ significantly from that of a stock market index. Two variables determine the shape of the return distribution function at the end of an extended period: the duration, T, of each certificate and the ratio, C/J, of cap to price of the underlying asset. By proper choice of these parameters an investor will find the investment that suits his or her individual risk profile best.

We assume a total period of 16 years during which discount certificates of varying T (0.5 years, 1 year, 2 years and 4 years) and different C/J ratios are iterated N times (32 times, 16 times, 8 times and 4 times for the certificate with the four-year term), so that N·T = 16. The underlying asset is a fictitious market index with a momentary price J = 1000, an expected yield of 4% p.a. and an average annual volatility of 20%. Its future behavior is simulated by the Black-Scholes model. The risk-free interest rate is set to 1% p.a.. For the purchase price of each certificate we take the price to be $K_{fair}$ which can be determined by the above quantities (see **Theoretical description of discount certificates and their iteration** (in German)).

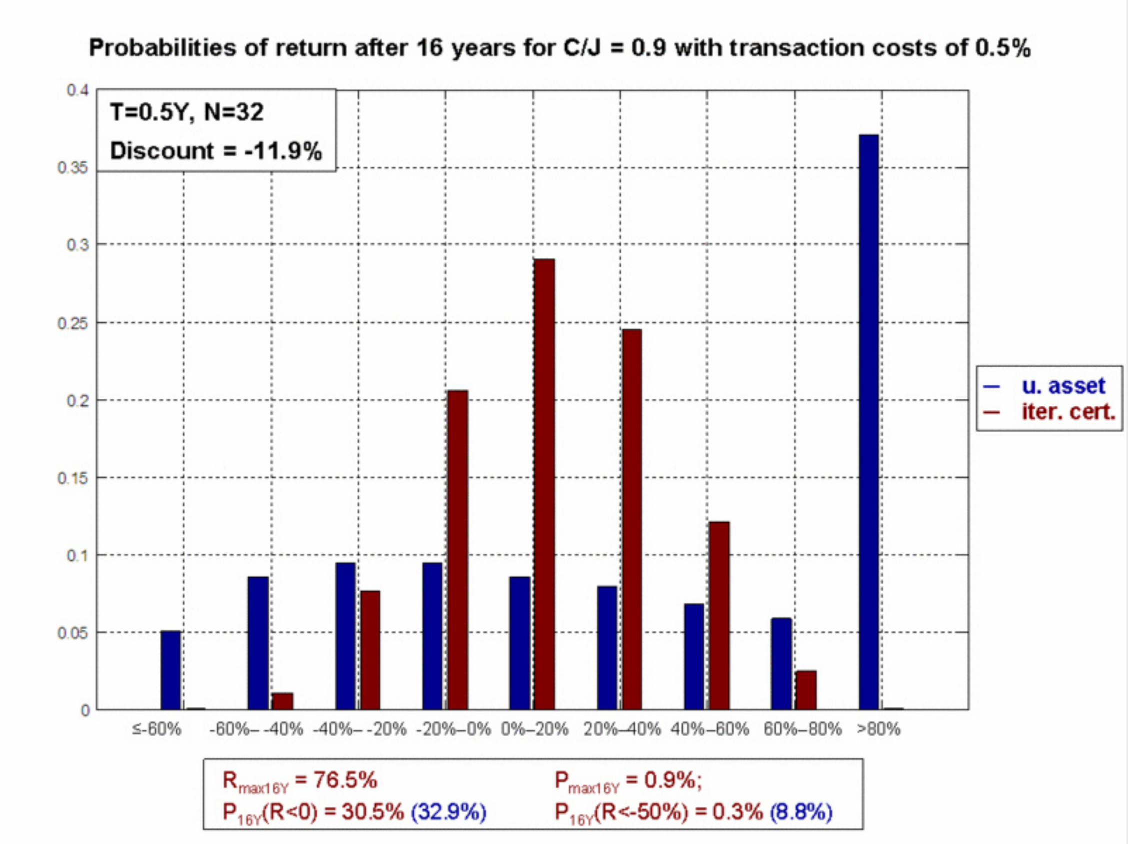
Taking C/J = 0.9 and C/J = 1 as examples, the following graphs show the probabilities of getting a certain range of returns after 16 years, **including fees** of 0.5% per transaction. Other relevant statistical quantities are the probability, $P_{max16Y}$, to achieve the maximum possible return, $R_{max16Y}$, after 16 years, as well as the loss probabilities, $P_{16Y}(R < 0)$ and $P_{16Y}(R < -50\%)$, for any loss or losses higher than -50% . The discount indicates the relative distance of the fair price from the underlying asset, i.e. $K_{fair}/J - 1$.

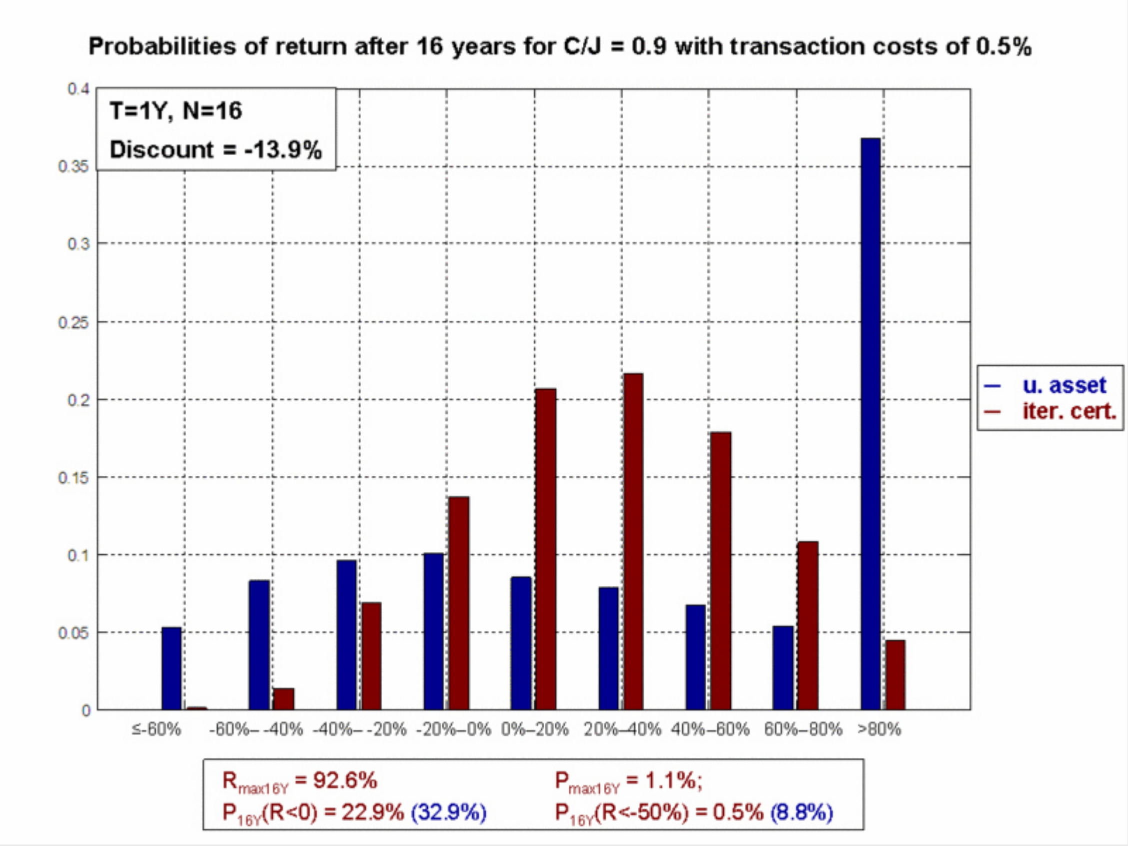
**Interpretation of the following graphs:** For example, if a six-month discount certificate with C/J = 0.9 is iterated 32-times, a return between 20% and 40% is achieved with a probability of nearly 25%. Note: The bar for returns >80% is so high, because this range of returns theoretically extends to infinity and all returns are "collected" therein, in contrast to all other ranges which are bounded.

# Discount Certificates

## 4. Iterations of discount certificates

### 1. Example: C/J = 0.9

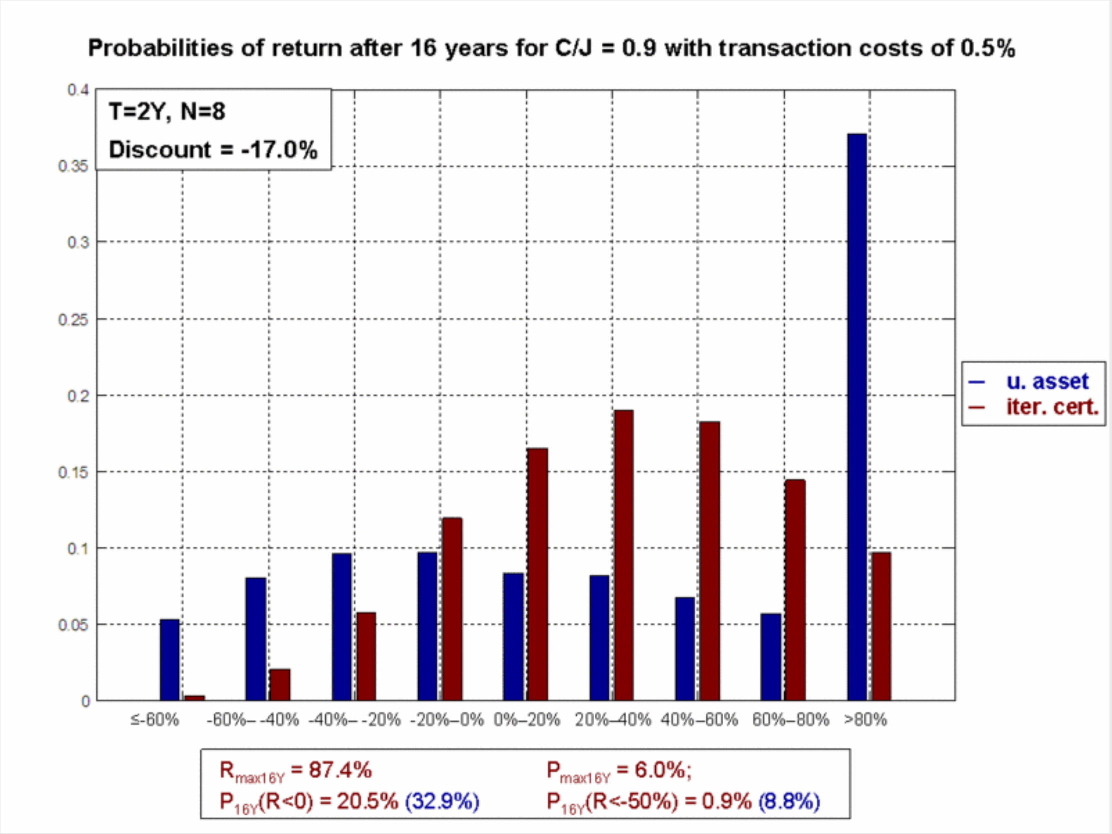


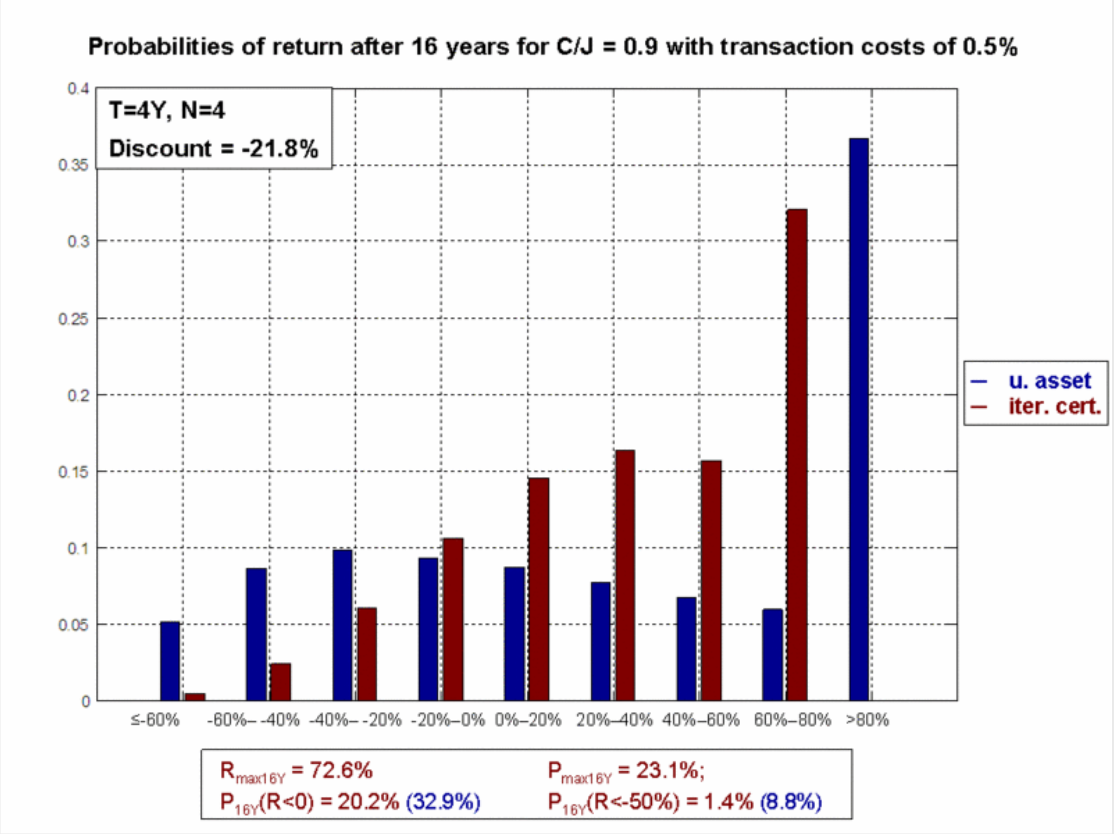



# Discount Certificates

## 4. Iterations of discount certificates

### 1. Example: C/J = 0.9

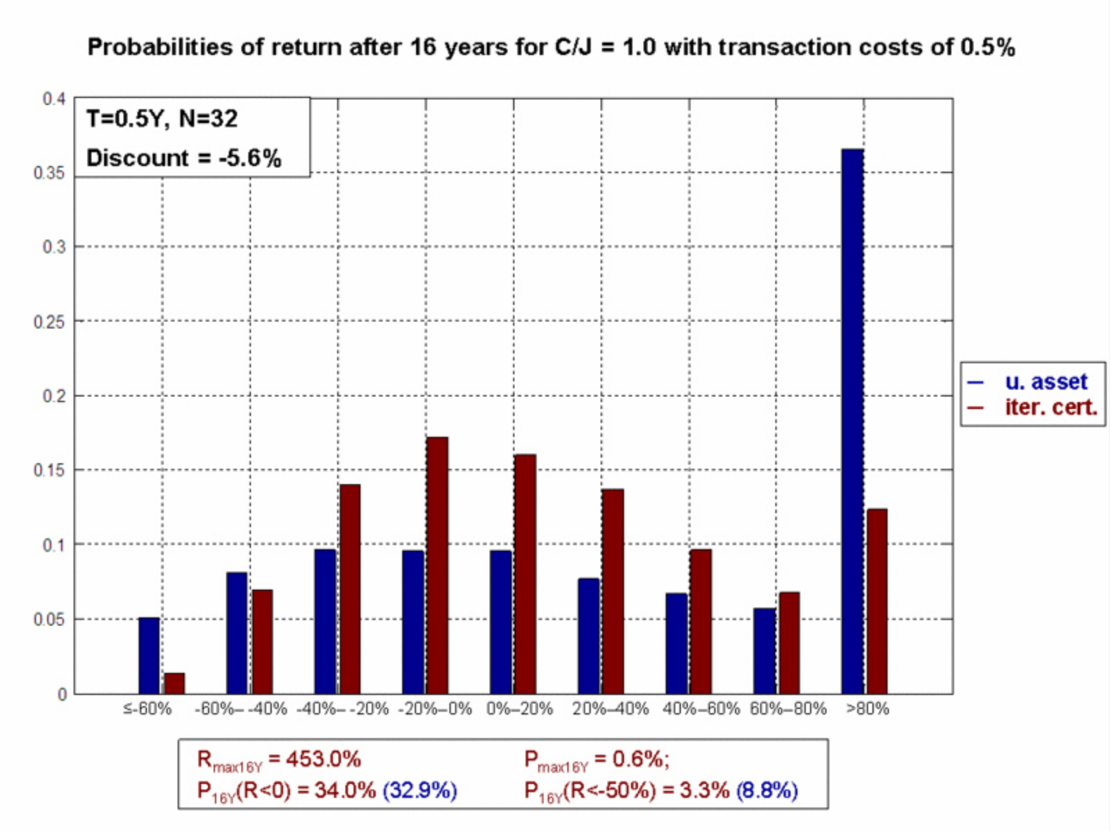


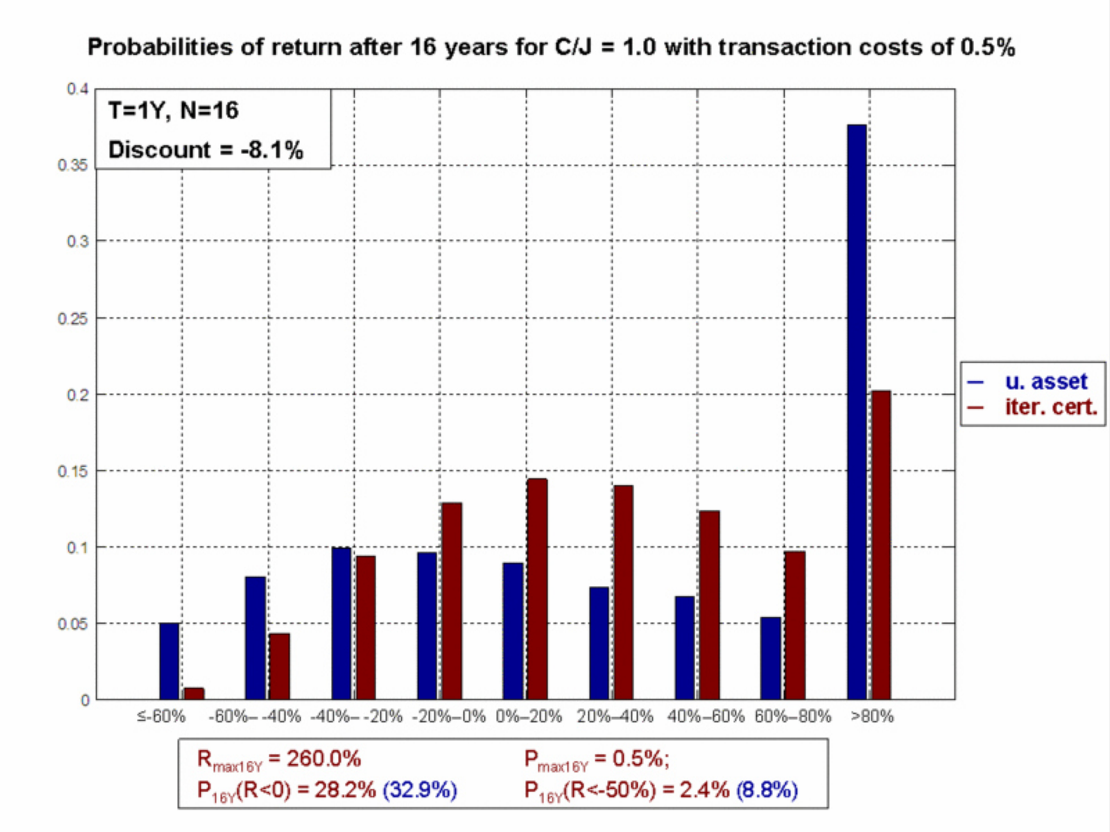



# Discount Certificates

## 4. Iterations of discount certificates

### 2. Example: C/J = 1

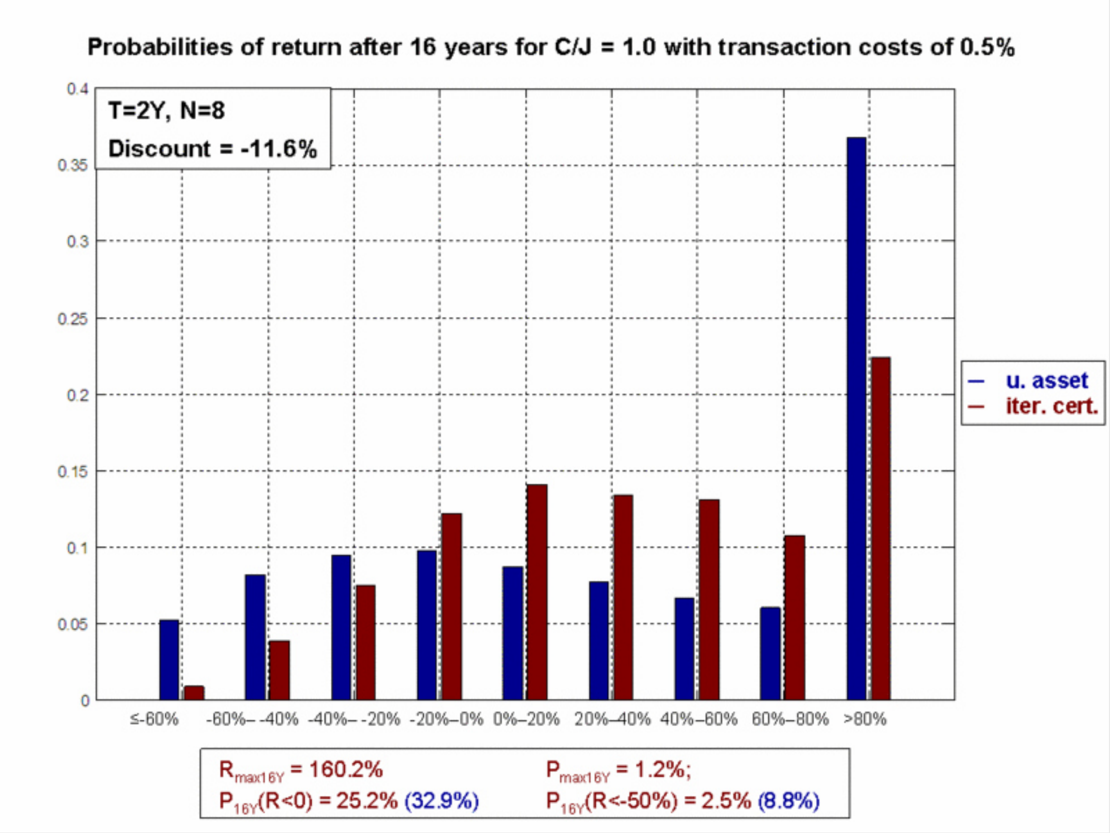


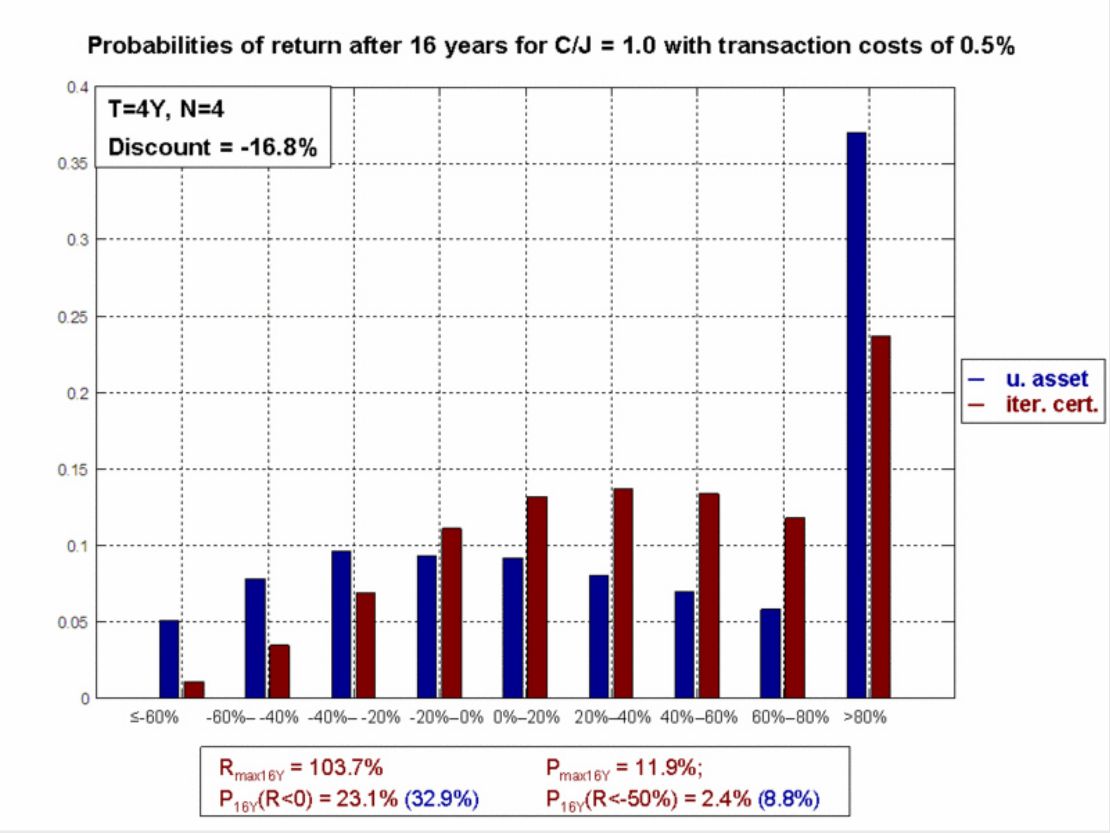



# Discount Certificates

## 4. Iterations of discount certificates

### 2. Example: C/J = 1

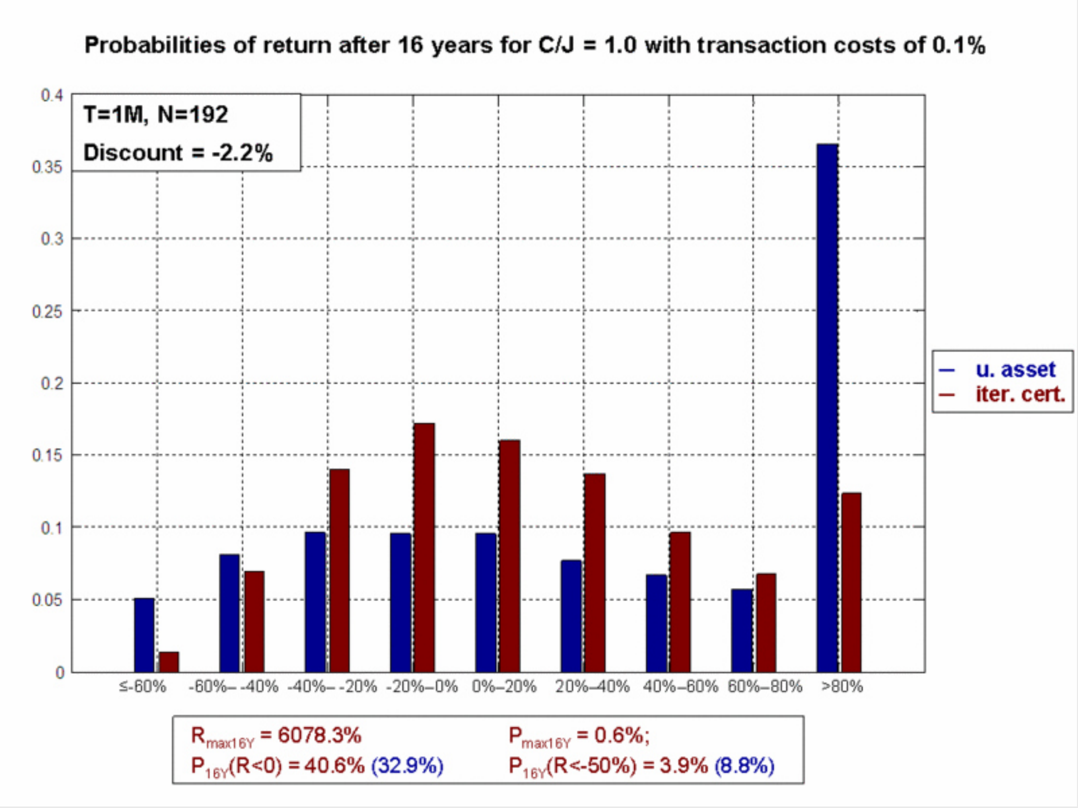


## Discount Certificates

**Banks offer rolling** (i.e. iterated) **discount certificates** that, in almost all cases, have a C/J close to 1 and a very short duration T, usually only 1-3 months. For these products, annual management fees up to 1.8% are charged. The following graph for C/J = 1 and a duration of one month shows the probabilities to get a certain range of returns after 16 years, if one assumes fees of 0.1% per transaction (corresponding to an annual management fee of 1.2% without additional transaction costs). When comparing the probabilities of return with, for example, those of an iterated certificate having T = 2Y, C/J = 0.9 and transaction costs of 0.5% (a variant that any investor can realize with only little effort by himself), one has to question the benefit of such products for investors: The loss probability of the rolling discount certificate is almost twice as high and the expected average return is about 30% less. The benefit for the banks, however, is evident.




**Results:**

- For zero fees there is a risk-return curve which is determined by C/J and the duration T. To stay close to this "optimal" curve the investor should choose long durations (T> 1 year) when fees are charged. The shorter the duration of a single discount certificate, the more important are low transaction costs and a fair price for the certificate, since otherwise the reductions of return can be significant for large numbers of iterations N.